Coisotropic D8-branes and model-building

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ABSTRACT: Up to now chiral type IIA vacua have been mostly based on intersecting D6-branes wrapping special Lagrangian 3-cycles on a $\mathbf{CY}_3$ manifold. We argue that there are additional BPS D-branes which have so far been neglected, and which seem to have interesting model-building features. They are coisotropic D8-branes, in the sense of Kapustin and Orlov. The D8-branes wrap 5-dimensional submanifolds of the $\mathbf{CY}_3$ which are trivial in homology, but contain a worldvolume flux that induces D6-brane charge on them. This induced D6-brane charge not only renders the D8-brane BPS, but also creates $D = 4$ chirality when two D8-branes intersect. We discuss in detail the case of a type IIA $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, where we provide explicit examples of coisotropic D8-branes. We study the chiral spectrum, SUSY conditions, and effective field theory of different systems of D8-branes in this orientifold, and show how the magnetic fluxes generate a superpotential for untwisted Kähler moduli. Finally, using both D6-branes and coisotropic D8-branes we construct new examples of MSSM-like type IIA vacua.

KEYWORDS: Compactification and String Models, D-branes, Superstring Vacua,
Intersecting branes models.

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Contents

1. Introduction

One of the most popular techniques to obtain $D = 4$ chiral string compactifications is the construction of type IIA orientifolds with D6-branes intersecting at angles [1, 2] (see [3] for reviews on this subject). The building blocks in these constructions are BPS D6-branes, wrapping 3-cycles corresponding to special Lagrangian submanifolds in a $\mathbf{CY}_3$. In this way

one can obtain semirealistic three generation models with a low-energy spectrum quite close to that of the MSSM. Remarkably simple and successful are the intersecting brane models based on the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold [4]. In particular, it was shown in [5] that a simple local set of D6-branes leading to a MSSM-like spectrum [6, 7] can be simply embedded into this orientifold background.

Here we would like to point out that, while the above type IIA picture is rather compelling, it is in fact far from complete. In particular, we will show that there are other BPS D-branes in these backgrounds, and whose model building applications have been ignored up to now. These are nothing but BPS D8-branes, wrapping coisotropic 5-cycles in the $\mathbf{CY}_3$ and with a non-trivial magnetic flux in their internal worldvolume.

Coisotropic A-branes were first introduced by Kapustin and Orlov [8] in the context of topological string theory, and seem to be required for a complete formulation of the Homological Mirror Symmetry conjecture. Here we will be interested in the particular case of D8's wrapping coisotropic 5-cycles in a $\mathbf{CY}_3$. That such Dp-brane exists may look surprising at first sight, since all 5-cycles in a generic $\mathbf{CY}_3$ are homologically trivial and one would not expect a stable object constructed out of them. However, as we will discuss in detail, coisotropic D8-branes are not only stable but also BPS, and this is because the magnetic flux on the D8-brane induces a non-trivial D6-brane charge on its worldvolume.

Coisotropic D8-branes have several interesting properties. Chirality arises in D8-D8 or D8-D6 systems due to a mixture of intersecting/magnetization chirality mechanisms. Because of this, Yukawa couplings among chiral fields are generated by a combination of wavefunction overlapping and open string world-sheet instantons. In addition, the BPS conditions on the coisotropic branes give rise to constraints which in the effective field theory may be interpreted as F and D-term cancellation. We will explicitly compute such F and D-terms in the particular case of the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold, where most of our discussion will be based. Unlike in the case of D6-branes at angles we will see that the F-term generated for D8-branes is non-trivial, and that it involves the untwisted Kähler moduli of the compactification.

From the model-building point of view these D8-branes also present a number of advantages over analogous models with only D6-branes at angles. In particular, the D6-charge carried by coisotropic D8-branes is not of the form (3-cycle) = (1-cycle) × (1-cycle) × (1-cycle), but rather a sum of two of these. This adds flexibility to the model-building and allows for possibilities not available with standard D6-branes, as we will show by means of explicit examples. Finally, the presence of D8-brane generates a superpotential for the Kähler moduli and some open string moduli of our compactification, without the need of closed string NSNS or RR fluxes. Notice that the constructions here discussed are CFT's, unlike the compactifications in which closed string fluxes are added.

The structure of the paper is as follows. In the next section the notion of coisotropic Dp-branes is reviewed and we argue why there should exist coisotropic D8-branes in generic $\mathbf{CY}_3$'s. In section 3 we discuss in detail the cases where our compact manifold is $\mathbf{T}^6$ and its $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, constructing explicit examples of BPS D8-branes on both backgrounds. We analyze the constraints coming from the BPS conditions of such D-branes and derive the RR tadpole cancellation conditions. In section 4 we discuss how chiral fermions ap-

pear at the overlap of D8-branes with some other D8 or D6-branes, and show that their multiplicity can be computed as the intersection number of two 3-cycles. Some aspects of the $\mathcal{N} = 1$ effective field theory of coisotropic D8-branes are examined in section 5, including a discussion on the gauge kinetic function, F- and D-terms, massive U(1)'s and Yukawa couplings. Section 6 illustrates possible model-building applications of coisotropic D8-branes. In particular we present two 3-generation MSSM-like models based on both coisotropic D8-branes and intersecting D6-branes. Finally, in section 7 we analyze the T-duals of the present constructions, and in particular the mirror type I vacua where coisotropic D8-branes become tilted D5-branes or D9-branes with off-diagonal magnetic fluxes. Some final comments are left for section 8, whereas in appendix A we briefly review the formal definition of coisotropic branes.

2. Coisotropic D8-branes

Let us consider type IIA string theory compactified on a Calabi-Yau three-fold $\mathcal{M}_6$. If we are interested in obtaining a $D = 4$ semi-realistic effective theory, we need to embed the Standard Model gauge group and matter content in our construction. This cannot be achieved by just considering closed strings [9], but we need to include space-filling D-branes in this setup. If, in addition, we aim to construct an $\mathcal{N} = 1$ effective field theory, we need to consider BPS, space-filling D-branes.

In principle, type IIA theory contains three different kinds of D-branes which may be both space-filling and BPS: D4, D6 and D8-branes. However, if our compactification manifold $\mathcal{M}_6$ has proper SU(3) holonomy, one would never think of obtaining a BPS D-brane out of a space-filling D4-brane. The reason is that for such manifolds $b_1 = 0$, and then our D4-brane (which is wrapping a 1-cycle of $\mathcal{M}_6$) would couple to a RR-potential C_5 without zero modes. This implies that the Chern-Simons action for a space-filling D4-brane vanishes, and so D4-branes wrapped on 1-cycles cannot carry any central charge. In practice, this is summarized by stating that D4-branes wrap 1-cycles which are 'homologically trivial'.[1]

The situation is quite different for D6-branes wrapping 3-cycles of $\mathcal{M}_6$. Here, because $b_3(\mathcal{M}_6) \neq 0$, the Calabi-Yau background gives rise to a plethora of central charges, which are classified by the elements of $H^3(\mathcal{M}_6, \mathbb{R})$. The BPS conditions for such D6-brane can be expressed in terms of the holomorphic and Kähler forms Ω and J [10], and read

$$\Omega = \frac{\sqrt{|G+\mathcal{F}|}}{\sqrt{|G|}} \, d\text{vol}_3$$
$$\mathcal{F} + iJ = 0 \tag{2.1}$$

where the background forms Ω, J and the metric G are pull-backed to the 3-cycle Π_3 wrapped by the D6-brane. In the language of special holonomy manifolds, these two conditions mean that Π_3 is a special Lagrangian submanifold of $\mathcal{M}_6$, with a flat bundle $\mathcal{F} = 2\pi\alpha' F + B$. Finally, from the point of view of the $D{=}4$ gauge theory arising from the

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[1]Such terminology is somewhat misleading, because a D4-brane could still be wrapping a torsional 1-cycle of $\mathcal{M}_6$.

D6-brane, these BPS conditions can be rephrased as D-flatness and F-flatness equations. More precisely

$$
\begin{aligned}
\text{D} - \text{flatness} && \text{Im}\,\Omega &= 0 \\
\text{F} - \text{flatness} && \mathcal{F} + iJ &= 0
\end{aligned}
\tag{2.2}
$$

One would naively say that D6-branes wrapping special Lagrangian 3-cycles exhaust all the possibilities of space-filling BPS D-branes. Indeed, we have already seen that we cannot construct them from D4-branes. A similar argument seems to hold for D8-branes, which also wrap 5-cycles $\Pi_5 \subset \mathcal{M}_6$ trivial in homology. However, this is not quite true because, unlike the D4-brane, a D8-brane is allowed to carry a non-trivial gauge bundle F without breaking Poincaré invariance. This gauge bundle modifies the Chern-Simons action of the D8-brane and, in particular, induces a D6-brane charge on its worldvolume [11] (see also [12]). If such D6-brane charge corresponds to a non-trivial element of $H^3(\mathcal{M}_6, \mathbb{R})$, then we can have a non-trivial Chern-Simons action via the coupling

$$
\int_{M_4 \times \Pi_5} \mathcal{F} \wedge C_7
\tag{2.3}
$$

and thus our D8-brane will have a non-trivial central charge, again related to $H^3(\mathcal{M}_6, \mathbb{R})$. Hence, we should be able to find a BPS stable object.

Given our past experience with type IIA orientifold vacua it may seem quite striking that, besides D6-branes wrapping special Lagrangians, there could also exist stable D8-branes which are mutually BPS with the former. However, this possibility has already arisen in the quite different context of topological string theory. Indeed, as described in [13] a D-brane wrapping a special Lagrangian (or rather just Lagrangian) n-cycle of a Calabi-Yau n-fold is the prototypical example of D-brane in the topological A-model, usually dubbed as A-brane. Naively, these are the only boundary conditions which are allowed for open strings in the A-model but, as shown by Kapustin and Orlov in [8], this is in fact not true. An A-brane does not necessarily wrap a Lagrangian n-cycle, but can also wrap a coisotropic $(n + 2k)$-cycle, $k = 1, \ldots, [n/2]$, if the appropriate worldvolume bundle F is introduced.[2] As emphasized by the authors, this fact proves essential in order to understand the full spectrum of D-branes in our theory and, in more theoretical grounds, to correctly formulate the Homological Mirror Symmetry conjecture.[3]

All this suggests that our previous BPS D8-brane should in fact correspond to a coisotropic D8-brane, in the sense of Kapustin and Orlov. Let us further motivate this by analyzing the supersymmetry conditions for coisotropic D-branes, which can be obtained either by a worldsheet [8, 15] or a worldvolume approach [16]. In the case of a D8-brane on a Calabi-Yau three-fold they can be written as

$$
\begin{aligned}
\mathcal{F} \wedge \Omega &= \frac{\sqrt{|G + \mathcal{F}|}}{\sqrt{|G|}}\, d\text{vol}_5 \\
(\mathcal{F} + iJ)^2 &= 0
\end{aligned}
\tag{2.4}
$$

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[2]See the appendix A for a mathematical definition of coisotropic submanifold.

[3]See [14] for a recent perspective of this problem in terms of pure spinors.

where Ω, J and G are now pull-backed over the 5-cycle Π_5. Notice that, when written in this way, the BPS conditions for D6 and D8-branes look extremely similar.

The D8-brane BPS equations take a particularly suggestive form when the pull-back of B vanishes identically, and then $\mathcal{F} = 2\pi\alpha' F$ defines a homology class of 3-cycles $[\Pi_3^F]$ via Poincaré duality in Π_5. This homology class is nothing but the D6-brane charge induced by the Chern-Simons coupling (2.3), which can now be written as

$$\mu_8 \int_{M_4 \times \Pi_5} 2\pi\alpha' F \wedge C_7 = \mu_6 \int_{M_4 \times \Pi_3^F} C_7 \tag{2.5}$$

where $\mu_p = (2\pi)^{-p}\alpha'^{-\frac{p+1}{2}}$, and Π_3^F is any representative of $[\Pi_3^F]$ (the Poincaré dual of $[F/2\pi]$). Because $[\Pi_3^F]$ is the central charge carried by our D8-brane, it should match the D8-brane tension in the BPS limit. This is indeed the case, since by integrating the first equation in (2.4) we obtain

$$T_{D8} = \mu_8 \int_{M_4 \times \Pi_5} 2\pi\alpha' F \wedge \Omega = \mu_6 \int_{M_4 \times \Pi_3^F} \Omega \tag{2.6}$$

and so the whole tension of the D8-brane equals that of a D6-brane wrapping a special Lagrangian in $[\Pi_3^F]$, matching the r.h.s. of eq. (2.5).

Regarding the second supersymmetry condition, it implies that $F \wedge J = 0$, and hence that Π_3^F cannot contain any holomorphic non-vanishing 2-cycle. This is automatic for Lagrangian 3-cycles, which supports the idea that Π_3^F could be seen as a special Lagrangian 3-cycle in Π_5. In addition, (2.4) implies that $\mathcal{F}^2 = J^2$ on Π_5, and this suggests that the D8-brane charge and the induced D4-brane charge of a coisotropic D8-brane, measured as

$$\mu_8 \int_{M_4 \times \Pi_5} C_9 \quad \text{and} \quad \mu_8 \int_{M_4 \times \Pi_5} \mathcal{F}^2 \wedge C_5 \tag{2.7}$$

need to be equal in magnitude.

Finally, we can again rewrite our BPS conditions in terms of $D = 4$ D-flatness and F-flatness conditions, namely [15, 17]

$$\begin{array}{ll} \text{D} - \text{flatness} & \text{Im}\,(\mathcal{F} \wedge \Omega) = 0 \\ \text{F} - \text{flatness} & (\mathcal{F} + iJ)^2 = 0 \end{array} \tag{2.8}$$

While the D-flatness condition of an A-brane is exact at all orders in α', in general we would expect to receive α' corrections to the F-flatness condition. This is indeed the case for special Lagrangian D6-branes, where the corrections arise from open string world-sheet instantons [18]. Although these corrections are less understood in the case of coisotropic A-branes, a proposal in terms of generalized Floer homology has been given in [19].

To summarize, we have argued that D6-branes are not the only space-filling BPS objects in type IIA Calabi-Yau compactifications, and that BPS D8-branes can also exist when endowed with the appropriate worldvolume flux F. We have identified such D8-branes as coisotropic A-branes, and have used their supersymmetry conditions to further motivate our initial intuition. Even in the coisotropic D-brane literature, the fact that

$b_5 = 0$ in most Calabi-Yau manifolds has led to believe that stable A-branes of this kind do not exist. However, as our previous discussion shows, the fact that Π_5 is homologically trivial should not be an obstruction to construct a BPS object. This can still be achieved if the D-brane charge induced by F is non-trivial in the homology of $\mathcal{M}_6$. In the next section, we will give further evidence of this general picture, by explicitly constructing BPS D8-branes in Calabi-Yau manifolds with proper SU(3) holonomy.

3. Coisotropic D8-branes in type IIA orientifolds

In this section we provide explicit examples of coisotropic D8-branes. This will not only illustrate the discussion above, but also prepare the ground to build $\mathcal{N} = 1$ vacua based on D8-branes. We will consider two different backgrounds, which are $\mathbf{T}^6$ and $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$. Eventually, we will also need to cancel the D8-brane charge and tension without breaking supersymmetry. This can be achieved by introducing O6-planes in our compactification, that is by applying the usual type IIA orientifold projection on the above backgrounds.

3.1 D8-branes on $\mathbf{T}^6$

There are not many examples of coisotropic D-branes in the literature. Most of the constructions are based on compactifications on $\mathbf{T}^4$ [8, 20] and $\mathbf{K3}$ [21]. Because these are 4-dimensional manifolds, the coisotropic D-brane wraps the whole manifold and hence a non-trivial 4-cycle. The situation is more involved for a D8-brane on a Calabi-Yau three-fold, because in general all the 5-cycles on $\mathcal{M}_6$ will be trivial. Two exceptions are $\mathbf{T}^6$ and $\mathbf{T}^2 \times \mathbf{K3}$, where coisotropic D8-branes are known to exist. Again, the known examples are very few, so one of our goals will be to classify the set of coisotropic D8-branes in these backgrounds. We will now proceed to analyze the case of $\mathbf{T}^6$, while $\mathbf{T}^2 \times \mathbf{K3}$ will arise later on, when considering D8-branes on $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$.

Let us then consider type IIA theory compactified on a toroidal background of the form $(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3$, where each two-torus has a rectangular shape. This family of manifolds is parameterized by six compactification radii, which can be arranged as three Kähler and three complex structure moduli. The complex structure moduli $i\tau_j$ are pure imaginary quantities, and they enter into the holomorphic 3-form as

$$\Omega = (4\pi^2\alpha')^{3/2} \prod_j \left(\frac{\operatorname{Re} T_j}{\tau_j}\right)^{1/2} dz^1 \wedge dz^2 \wedge dz^3 \tag{3.1}$$

$$dz^j = dx^j + i\tau_j dy^j \tag{3.2}$$

where $x^j \sim x^j + 1$ and $y^j \sim y^j + 1$ are periodic coordinates, and $\operatorname{Re} T_j$ is the area of the jth $\mathbf{T}^2$, in units of $4\pi^2\alpha'$. This Kähler modulus is complexified by including the background B-field on $(\mathbf{T}^2)_j$, so that the combination $4\pi^2\alpha' \, T_j = A_j + iB_j$ enters the complexified Kähler form J_c as[4]

$$J_c = B + iJ = 4\pi^2\alpha' \sum_{j=1}^3 \frac{T_j}{2\tau^j} \, dz^j \wedge d\bar{z}^j = i \, 4\pi^2\alpha' \sum_{j=1}^3 T_j \, dy^j \wedge dx^j. \tag{3.3}$$

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[4]In our conventions $\int_{\mathbf{T}^6} dx^1 \wedge dx^2 \wedge dx^3 \wedge dy^1 \wedge dy^2 \wedge dy^3 = 1$, so that J^3 measures the volume of $\mathbf{T}^6$.
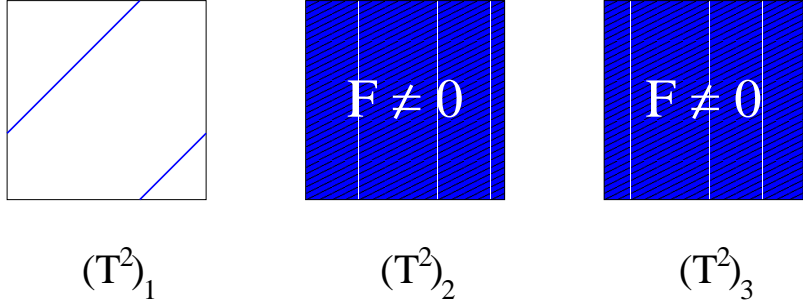
Figure 1: Coisotropic D8-brane on $\mathbf{T}^6$.

Of course, this is just a subspace of the whole $\mathbf{T}^6$ moduli space but, once the $\mathbb{Z}_2 \times \mathbb{Z}_2$ and orientifold projection are implemented, any other $\mathbf{T}^6$ modulus will be projected out. We will hence restrict ourselves to the above setup.

Any 5-cycle of our compactification is of the form $\Pi_5 = \mathbf{T}^5 = \mathbf{S}^1 \times \mathbf{T}^4$, so our D8-brane will be wrapping two complex dimensions and a 1-cycle in the third complex dimension. We will denote such 5-cycle as

$$\Pi_5 = (n^i, m^i)_i \times (\mathbf{T}^2)_j \times (\mathbf{T}^2)_k, \qquad i \neq j \neq k \neq i \tag{3.4}$$

where $n^i, m^i \in \mathbb{Z}$ are the D8-brane wrapping numbers along the directions x^i and y^i, respectively, and $\{ijk\}$ are in cyclic permutation of $\{123\}$. We now construct a coisotropic D8-brane by introducing a non-trivial gauge bundle $F = dA$ on the four-torus $(\mathbf{T}^2)_j \times (\mathbf{T}^2)_k$, such that the supersymmetry conditions (2.4) are satisfied (see figure 1). For instance, let us a consider a single D8-brane such that

$$\begin{aligned} \Pi_5 &= (1,0)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3 \\ F/2\pi &= dx^2 \wedge dx^3 - dy^2 \wedge dy^3 \end{aligned} \tag{3.5}$$

A simple computation gives

$$\begin{aligned} F^2 &= 4\pi^2 \, dx^2 \wedge dy^2 \wedge dx^3 \wedge dy^3 \\ J_c \wedge F &= 0 \\ J_c^2 &= -(4\pi^2 \alpha')^2 \, T_2 T_3 \, dx^2 \wedge dy^2 \wedge dx^3 \wedge dy^3 \end{aligned} \tag{3.6}$$

where all powers of J_c have been pull-backed to Π_5. The second supersymmetry condition in (2.4) then reads

$$(\mathcal{F} + iJ)^2 = (2\pi\alpha' F + J_c)^2 = 0 \quad \Longleftrightarrow \quad T_2 T_3 = 1 \tag{3.7}$$

On the other hand we have that

$$\mathcal{F} \wedge \Omega = (4\pi^2 \alpha')^{5/2} \prod_j \left(\frac{\operatorname{Re} T_j}{\tau_j} \right)^{1/2} (1 + \tau_2 \tau_3) \frac{d\mathrm{vol}_{\Pi_5}}{\sqrt{G}} \tag{3.8}$$

where again Ω and the metric G are pull-backed to Π_5. This quantity needs to be proportional to the D8-brane DBI action, whose square is given by

$$
\begin{aligned}
\det (G + \mathcal{F}) &= (4\pi^2 \alpha')^5 \frac{\mathrm{Re}\, T_1}{\tau_1} \left[|T_2 T_3 - 1|^2 + \frac{\mathrm{Re}\, T_2 \mathrm{Re}\, T_3}{\tau_2 \tau_3} (1 + \tau_2 \tau_3)^2 \right] \\
&= (4\pi^2 \alpha')^5 \prod_j \frac{\mathrm{Re}\, T_j}{\tau_j} (1 + \tau_2 \tau_3)^2
\end{aligned}
\tag{3.9}
$$

where in the second line we have made use of (3.7). Thus, by taking the square root of (3.9) we recover the first supersymmetry condition in (2.4) for any values of τ_1, τ_2 and τ_3. Alternatively, one can see that (3.8) is always a real quantity, and hence the D-flatness condition in (2.8) is trivial.

How can we interpret these results? The fact that supersymmetry restricts the Kähler moduli is not surprising. As explained above, coisotropic D-branes are BPS because of their non-trivial worldvolume flux F. If we take our internal manifold to the decompactification limit, F will be extremely diluted and we will approach the limit $F = 0$, where the D-brane is not BPS.[5] Hence the supersymmetry conditions of a coisotropic D-brane should constrain the Kähler moduli of the compactification. As will be discussed in section 5, this can be understood in terms of a D-brane generated superpotential.

Notice that our coisotropic D8-brane (3.5) factorizes as a product of a 1-cycle on $\mathbf{T}^2$ and a 4-dimensional coisotropic D-brane on $\mathbf{T}^4$:

$$
\begin{aligned}
\Pi_4 &= (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3 \\
F/2\pi &= dx^2 \wedge dx^3 - dy^2 \wedge dy^3
\end{aligned}
\tag{3.10}
$$

Actually, such example of $\mathbf{T}^4$ coisotropic D-brane was already given in [8], and analyzed in great detail in [20]. There is also a **K3** analogue of (3.10), dubbed in [21] as *canonical coisotropic* D-brane. An interesting fact is that the D-brane charge induced by F, given by its Poincaré dual on $\mathbf{T}^4$, is

$$
[\Pi_2^F] = [(1,0)_2 \times (1,0)_3] + [(0,1)_2 \times (0,-1)_3]
\tag{3.11}
$$

This is not the homology class of any factorizable 2-cycle $(n^2, m^2) \times (n^3, m^3)$ considered in the intersecting D-brane framework, but rather a sum of two of them. These homology classes which are not factorizable as a product of 1-cycles arise in intersecting D-brane models when we consider two D-branes that have been through a recombination process [22, 23]. However, such recombined D-branes are not exact boundary states of the theory and hence it is much harder to analyze their physics (or even to prove their existence) than in the usual factorizable case. On the other hand, the coisotropic D-brane will naturally carry these non-factorizable charges, while it can still be described as an exact CFT boundary state.

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[5]More precisely, this statement is only true for proper SU(3) holonomy manifolds like $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$. This is not the case of $\mathbf{T}^6$, where we have a extended bulk supersymmetry and a D8-brane on (3.4) with $F = 0$ is always BPS. The correct statement is then that such D8-brane does not preserve the same supersymmetry as a D6-brane at angles.

In fact, all these observations made for the example (3.5) hold in general. First, any coisotropic D8-brane on $\mathbf{T}^6$ can be seen as a product of a 1-cycle and a coisotropic D-brane on $\mathbf{T}^4$. Second, the worldvolume flux F on this $\mathbf{T}^4$ induces a D6-brane charge of the form

$$[\Pi_3^F] = [(n^i, m^i)_i] \otimes [\Pi_2^{\mathrm{jk}}] \tag{3.12}$$

where Π_2^{jk} is a Lagrangian, non-factorizable 2-cycle on $(\mathbf{T}^2)_j \times (\mathbf{T}^2)_k$. In fact, any non-factorizable 2-cycle can be decomposed as a sum of two factorizable 2-cycles [24], so we finally have that

$$\begin{aligned}
[\Pi_3^F] &= [(n^i, m^i)_i] \otimes [\Pi_2^\alpha + \Pi_2^\beta] \\
&= [(n^i, m^i)_i] \otimes [(n_\alpha^j, m_\alpha^j)_j (n_\alpha^k, m_\alpha^k)_k + (n_\beta^j, m_\beta^j)_j (n_\beta^k, m_\beta^k)_k]
\end{aligned} \tag{3.13}$$

where the intersection number $I_{\alpha\beta}^{\mathbf{T}^4} = -(n_\alpha^j m_\beta^j - n_\beta^j m_\alpha^j)(n_\alpha^k m_\beta^k - n_\beta^k m_\alpha^k)$ is nothing but $\int_{(\mathbf{T}^2)_j \times (\mathbf{T}^2)_k} F^2/4\pi^2$, and so it cannot vanish. Finally, such coisotropic D8-brane is made up of a flat submanifold and a constant worldvolume flux, so it can be described by an exact boundary state.

An advantage of this intuitive picture is that it helps producing some new examples of coisotropic D8-branes. Indeed, we just need to choose the ten wrapping numbers above such that, for some choice of complex structure τ_i, the two factorizable 3-cycles in (3.13) are mutually supersymmetric, or

$$\begin{aligned}
\mathrm{Arg}\left(\int_{(n^i, m^i)} dz^i\right) &= \theta_1 \\
\mathrm{Arg}\left(\int_{\Pi_2^\alpha} dz^j \wedge dz^k\right) &= \mathrm{Arg}\left(\int_{\Pi_2^\beta} dz^j \wedge dz^k\right) = \theta_2
\end{aligned} \qquad \theta_1 + \theta_2 = 0 \bmod 2\pi \tag{3.14}$$

where again $\{ijk\}$ is a cyclic permutation of $\{123\}$. By taking $F/2\pi$ as the Poincaré dual of (3.13) in Π_5 we would get a coisotropic D8-brane satisfying the D-flatness condition in (2.8), and by imposing

$$T_j \cdot T_k = I_{\alpha\beta}^{\mathbf{T}^4} \tag{3.15}$$

we will also satisfy the F-flatness condition.

On the other hand, the decomposition of a non-factorizable 2-cycle is not unique, and so the wrapping numbers in (3.13) are not all independent. In addition, the conditions (3.14) are stronger than the D-flatness condition, and one can find examples of BPS D8-branes that cannot be written in such form. It turns then more useful to describe a D8-brane directly in terms of the 5-cycle (3.4) and a general constant $\mathrm{U}(1)$ bundle F as

$$\begin{aligned}
\Pi_5 &= (n^i, m^i)_i \times (\mathbf{T}^2)_j \times (\mathbf{T}^2)_k, \\
F/2\pi &= n^{xx} dx^j \wedge dx^k + n^{xy} dx^j \wedge dy^k + n^{yx} dy^j \wedge dx^k + n^{yy} dy^j \wedge dy^k \\
&\quad + m^{xy,j} dx^j \wedge dy^j + m^{xy,k} dx^k \wedge dy^k
\end{aligned} \tag{3.16}$$

where all the n's and m's are integer numbers, and $\{ijk\}$ is again a cyclic permutation of $\{123\}$. It is easy to see that the F-flatness condition now reads

$$(T_j + im^{xy,j}) \cdot (T_k + im^{xy,k}) = n^{xy} n^{yx} - n^{xx} n^{yy} \tag{3.17}$$

and so the only effect of $m^{xy,j}$, $m^{xy,k}$ is to shift the B-field by an integer number. For this reason we will set $m^{xy,j} = m^{xy,k} = 0$ from now on, and describe a coisotropic D8-brane a in terms of the six integer numbers

$$\text{D8}_a \ : \ (n_a^i, m_a^i)_i \times (n_a^{xx}, n_a^{xy}, n_a^{yx}, n_a^{yy})_{\text{jk}} \tag{3.18}$$

defined by (3.16). Comparing with the description (3.13), we find the identifications

$$
\begin{aligned}
n^{yy} &= -(n_\alpha^j n_\alpha^k + n_\beta^j n_\beta^k) & n^{xx} &= -(m_\alpha^j m_\alpha^k + m_\beta^j m_\beta^k) \\
n^{xy} &= m_\alpha^j n_\alpha^k + m_\beta^j n_\beta^k & n^{yx} &= n_\alpha^j m_\alpha^k + n_\beta^j m_\beta^k
\end{aligned}
\tag{3.19}
$$

In the notation (3.18) we also have that the pull-back of $\mathcal{F} \wedge \Omega$ reduces to

$$\mathcal{F} \wedge \Omega \ = \ c \cdot (n^i + i\tau_i m^i)\,(n^{xx}\tau_j\tau_k + i n^{xy}\tau_j + i n^{yx}\tau_k - n^{yy})\, d\text{vol}_{\Pi_5} \tag{3.20}$$

where $c \in \mathbb{R}$, and hence the D-flatness condition reads

$$m^i n^{yy}\tau_i - n^i n^{xy}\tau_j - n^i n^{yx}\tau_k \ = \ m^i n^{xx}\tau_i\tau_j\tau_k \tag{3.21}$$

plus the additional condition $\text{Re}\, F \wedge \Omega > 0$. As we will see below, in the case of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold a coisotropic D8-brane can also be defined by the same set of integer numbers (3.18), and the chiral spectrum as well as the effective field theory quantities of a D8-brane configuration will only depend on them.

Finally, we are interested in introducing O6-planes in our theory. For this we mod out our toroidal compactification by the orientifold action $\Omega\mathcal{R}(-1)^{F_L}$, where Ω is the usual world-sheet parity, F_L stands for left-handed spacetime fermion number, and $\mathcal{R}$ is the antiholomorphic involution

$$\mathcal{R} \ : \ (z^1, z^2, z^3) \quad \mapsto \quad (\bar{z}^1, \bar{z}^2, \bar{z}^3). \tag{3.22}$$

In general, a magnetized D8-brane will not be left invariant under this orientifold action, so we need to include its orientifold image, given by[6]

$$\text{D8}_{a'} \ : \ (-n_a^i, m_a^i)_i \times (-n_a^{xx}, n_a^{xy}, n_a^{yx}, -n_a^{yy})_{jk} \tag{3.23}$$

Notice that in some cases like (3.5) a D8-brane is mapped by $\Omega\mathcal{R}(-1)^{F_L}$ to an anti-D8-brane, so the total D8-brane charge vanishes. Nevertheless, it is easy to check that if D8$_a$ satisfies the supersymmetry conditions (3.17) and (3.21) so will $D8_{a'}$, so both D-branes are BPS objects.

3.2 D8-branes on $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

The fact that we can construct coisotropic D8-branes in $\mathbf{T}^6$ is perhaps not surprising, given that in this compactification manifold $b_5 = 6$ and then D8-branes can wrap non-trivial 5-cycles. One may then wonder how a similar construction could be carried in general Calabi-Yau manifolds, where $b_5 = 0$ and any 5-cycle Π_5 will be trivial in homology.

---

[6]The orientifold action which takes D8$_a$ to D8$_{a'}$ may be not obvious at this stage, but is quite easy to understand from the mirror type I picture, which will be discussed in section 7.

$(i = 1)$	D8 charge	D6 charge	D4 charge
a	$[\Pi_5]$	$[\Pi_3^F]$	$[\Pi_1^{F^2}]$
ωa	$[\Pi_5]$	$[\Pi_3^F]$	$[\Pi_1^{F^2}]$
θa	$-[\Pi_5]$	$[\Pi_3^F]$	$-[\Pi_1^{F^2}]$
$\omega \theta a$	$-[\Pi_5]$	$[\Pi_3^F]$	$-[\Pi_1^{F^2}]$

Table 1: D-brane charges of a coisotropic D8-brane of the form (3.4), when embedded as a bulk D-brane on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold. Here $[\Pi_3^F]$ is defined as the Poincaré dual of $[F/2\pi]$ in Π_5, while $[\Pi_1^{F^2}]$ is the Poincaré dual of $[F^2/4\pi^2]$. This table holds for the case where $i = 1$ in (3.4), while in the cases $i = 2$ and $i = 3$, the roles of θ, ω and $\theta\omega$ are interchanged.

In order to get more intuition about this problem, we can consider manifolds close to $\mathbf{T}^6$ while carrying proper SU(3) holonomy. These are toroidal orbifolds of the form $\mathbf{T}^6/\Gamma$, where $\Gamma \subset$ SO(6) is a discrete subgroup of SU(3) but not of SU(2). A simple example is given by the orbifold $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, with the $\mathbb{Z}_2 \times \mathbb{Z}_2$ generators acting as

$$\theta : (z^1, z^2, z^3) \mapsto (-z^1, -z^2, z^3) \tag{3.24}$$

$$\omega : (z^1, z^2, z^3) \mapsto (z^1, -z^2, -z^3) \tag{3.25}$$

and where z^i is the complex coordinate associated to $(\mathbf{T}^2)_i$. Just as in [4], we will consider the choice of discrete torsion such that each $\mathbb{Z}_2 \times \mathbb{Z}_2$ fixed point contains a collapsed 2-cycle, and so the complete orbifold cohomology is given by $(h_{11}, h_{21}) = (51, 3)$.

Following the usual strategy for describing D-branes on orbifolds [25], we can construct a coisotropic D8-brane in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ by considering a D8-brane on the covering space $\mathbf{T}^6$, then adding its images under $\mathbb{Z}_2 \times \mathbb{Z}_2$, and finally quotienting our theory by the orbifold action. If the D8-brane is not placed on top of any fixed point and has non-trivial Wilson lines, all the images are different and we are dealing with a *bulk* D8-brane whose gauge group is U(1). Notice that this also applies to a D6-brane at angles, although there is one important difference. The homology class of a D6-brane $[\Pi_3] = [(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)]$ is invariant under the full orbifold action (3.24), (3.25), while this is not true for a D8-brane. For instance, in our $\mathbf{T}^6$ example (3.5), the homology class $[\Pi_5]$ is mapped into $-[\Pi_5]$ by the action of θ, while F is mapped to $-F$. On the other hand, the action of ω leaves these two quantities invariant. It is easy to see that one gets the same kind of behavior for any coisotropic D8-brane. This can be understood by the fact that a coisotropic D8-brane on $\mathbf{T}^6$ will carry D8, D6 and D4-brane charges, which can be described by homology classes of 5, 3 and 1-cycles on $\mathbf{T}^6$. When quotienting by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold group the non-trivial 5 and 1-cycles will be projected out, and so the total D8 and D4-brane charges (2.7) should vanish. On the contrary, the induced D6-brane charge (2.3) will survive the orbifold projection, as can be appreciated from table 1.

However, bulk coisotropic D8-branes are not very interesting from the model-building point of view. The reason is that, as we will show in the next section, the chiral spectrum of a coisotropic D8-brane only depends on the D6-brane charge that it carries. Bulk coisotropic D8-branes carry bulk D6-brane charges and, as implicit in [4], bulk D6-branes
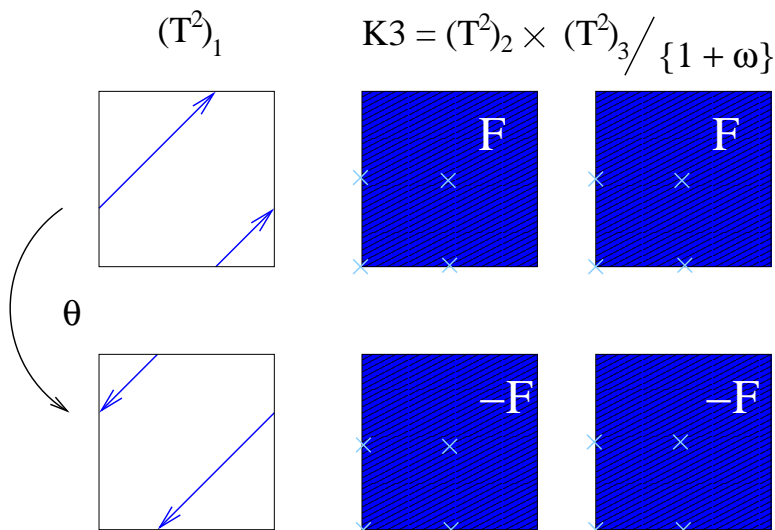
Figure 2: Fractional coisotropic D8-brane on $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$.

do only give rise to models with an even number of chiral families. Hence we would never achieve a three-family model out of bulk coisotropic D8-branes.

The way out is to consider D8-branes which are fractional with respect to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold action, and so they can carry fractional D6-brane charge. Recall that in this particular choice of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold fractional D6-branes are $\frac{1}{2}$ bulk D6-branes [26, 27], and so will be the case for fractional D8-branes. Indeed, from table 1 we see that only the action of one generator of $\mathbb{Z}_2 \times \mathbb{Z}_2$ leaves a D8-brane invariant, while the other maps it to its anti-D8-brane. Hence, a D8-brane can only be fractional with respect to a $\mathbb{Z}_2$ subgroup of $\mathbb{Z}_2 \times \mathbb{Z}_2$.

For concreteness, let us consider a D8-brane of the form (3.4) and with $i = 1$, so that it is left invariant by the action of ω. Generically, such D8-brane will not be on top of any θ-fixed point in $(\mathbf{T}^2)_1$, and so locally it will not know about the action of θ. It can thus be described by a 5-cycle of the form (see figure 2)

$$\Pi_5 = (n^1, m^1) \times \mathbf{T}^4/\mathbb{Z}_2^\omega \subset \mathbf{T}^2 \times \mathbf{T}^4/\mathbb{Z}_2^\omega \simeq \mathbf{T}^2 \times \mathbf{K3} \tag{3.26}$$

where $\mathbb{Z}_2^\omega$ stands for the $\mathbb{Z}_2$ subgroup of $\mathbb{Z}_2 \times \mathbb{Z}_2$ generated by ω, and so $\mathbf{T}^2 \times \mathbf{K3}$ can be seen as the covering space of $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ before quotienting by θ. We can then describe a coisotropic D8-brane in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ in terms of a D8-brane in $\mathbf{T}^2 \times \mathbf{K3}$ plus its image under θ. Moreover, because the worldvolume flux F is fully contained inside $\mathbf{K3}$, this D8-brane can be understood as a product of a 1-cycle in $\mathbf{T}^2$ and a coisotropic D-brane on $\mathbf{K3}$.

Let us now see which D6-brane charge is carried by a fractional D8-brane. The only difference with the toroidal case is that $[F/2\pi]$ is now an element of $H^2(\mathbf{K3}, \mathbb{Z})$, rather than of $H^2(\mathbf{T}^4, \mathbb{Z})$. As a result, the D6-brane charge is again of the form (3.12), but now Π_2 should be seen as a Lagrangian 2-cycle of $\mathbf{K3}$. As explained in [28], the difference between $H_2(\mathbf{T}^4, \mathbb{Z})$ and $H_2(\mathbf{K3}, \mathbb{Z})$ amounts to including fractional D-branes in the orbifold $\mathbf{T}^4/\mathbb{Z}_2$.

More precisely, $H_2(\mathbf{T}^4, \mathbb{Z})$ is a sublattice of $H_2(\mathbf{K3}, \mathbb{Z})$ which corresponds to bulk D-branes in $\mathbf{T}^4/\mathbb{Z}_2$. The elements of $H_2(\mathbf{K3}, \mathbb{Z})$ that are not in $H_2(\mathbf{T}^4, \mathbb{Z})$ contain the 'minimal' 2-cycles of $\mathbf{K3}$ and, in the orbifold limit $\mathbf{T}^4/\mathbb{Z}_2$, they have the form

$$\Pi_2^{\mathrm{fr}} = \frac{1}{2}\Pi_2^{\mathrm{b}} + \frac{1}{2}\left(\sum_{i,j \in \Pi_2^{\mathrm{b}}} \epsilon_{ij}\Pi_{ij}^{\omega}\right) \tag{3.27}$$

where Π_2^{b} is a bulk 2-cycle, inherited from the 2-cycles on $\mathbf{T}^4$ that survive the $\mathbb{Z}_2$ action of ω. Π_{ij}^{ω} stand for the 4×4 fixed points of $\mathbf{T}^4/\mathbb{Z}_2$ or, more precisely, the 16 collapsed $\mathbb{P}^1$'s that arise from such singularities. In general not all the collapsed 2-cycles contribute to (3.27), but only those which are crossed by Π_2^{b}. The phases $\epsilon_{ij} = \pm 1$ then correspond to the two possible orientations with which the two-cycle can wrap around the blown-up $\mathbb{P}^1$ labeled by ij. Because (3.27) contains the exceptional 2-cycles Π_{ij}^{ω}, a D-brane wrapped on it is stuck at the fixed points of $\mathbf{T}^4/\mathbb{Z}_2$, and in fact corresponds to a $\frac{1}{2}$ fractional D-brane of the $\mathbf{T}^4/\mathbb{Z}_2$ orbifold. Taking all the phases ϵ_{ij} positive corresponds to choosing one Chan-Paton phase, while taking them all negative corresponds to choosing the opposite one. The intermediate choices of ϵ_{ij}'s are constrained by consistency conditions, and they can be understood as discrete Wilson lines turned on. See, e.g., [28, 29] for more detailed discussions.

Given this description of a fractional D-brane it is now clear how to construct a coisotropic D8-brane with fractional D6-brane charge. We first consider a fractional D8-brane of the form (3.26) and then a pair of minimal 2-cycles $(\Pi_2^{\mathrm{fr}})^{\alpha}$ and $(\Pi_2^{\mathrm{fr}})^{\beta}$ such that (3.12)–(3.14) are satisfied. Just as in the toroidal case, our worldvolume flux F will be given by the Poincaré dual of $[(\Pi_2^{\mathrm{fr}})^{\alpha}] + [(\Pi_2^{\mathrm{fr}})^{\beta}]$ in Π_5, which means that $[F/2\pi]$ will be an element of $H^2(\mathbf{K3}, \mathbb{Z})$ but not of the sublattice $H^2(\mathbf{T}^4, \mathbb{Z})$. That is, F will be of the form

$$F^{\mathrm{fr}} = \frac{1}{2}F^{\mathrm{b}} + \frac{1}{2}F^{\mathrm{tw}} \tag{3.28}$$

where F^{b} is an element of $H^2(\mathbf{T}^4, \mathbb{Z})$ and F^{tw} is the Poincaré dual of a sum of exceptional 2-cycles $\sum_{ij} \epsilon_{ij}\Pi_{ij}$ in $\mathbf{K3}$ which, in the orbifold limit $\mathbf{T}^4/\mathbb{Z}_2$, can be associated to the twisted sectors of the theory. For simplicity, we will choose all the phases ϵ_{ij} to have the same sign, which amounts to not turning any discrete Wilson line.

The pair (Π_5, F^{fr}) is a coisotropic D8-brane which is fractional with respect to ω, but in order to have a well-defined object we still need to add its image under θ. From the toroidal case we know how Π_5 and F^{b} transform under the action of θ. The behavior of F^{tw} is more subtle and depends on the choice of discrete torsion. Recall that in a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold the choice of discrete torsion amounts to specifying the action of θ on the collapsed $\mathbb{P}^1$'s of $\mathbf{K3} \simeq \mathbf{T}^4/\mathbb{Z}_2^{\omega}$. The two possibilities are

$$\theta : \mathbb{P}^1 \mapsto \eta\mathbb{P}^1 \tag{3.29}$$

with $\eta = \pm 1$. In our case $\eta = 1$, i.e., the collapsed two cycles do not change orientation under the action of θ, and so the phases ϵ_{ij} in (3.27) do not change when we consider the

$(i = 1)$	D8 charge	D6 charge	tw. D6 charge	D4 charge
a	$[\Pi_5]$	$[\Pi_3^{F^{\mathrm{b}}}]$	$[\Pi_3^{F^{\mathrm{tw}}}]$	$[\Pi_1^{F^2}]$
θa	$-[\Pi_5]$	$[\Pi_3^{F^{\mathrm{b}}}]$	$-[\Pi_3^{F^{\mathrm{tw}}}]$	$-[\Pi_1^{F^2}]$
$\tilde{a}$	$[\Pi_5]$	$[\Pi_3^{F^{\mathrm{b}}}]$	$-[\Pi_3^{F^{\mathrm{tw}}}]$	$[\Pi_1^{F^2}]$
$\theta\tilde{a}$	$-[\Pi_5]$	$[\Pi_3^{F^{\mathrm{b}}}]$	$[\Pi_3^{F^{\mathrm{tw}}}]$	$-[\Pi_1^{F^2}]$

Table 2: D-brane charges of a fractional coisotropic D8-brane and its image under θ, given the choice of discrete torsion $\eta = 1$. The upper two rows represent a $\mathbb{Z}_2^\omega$-fractional D8-brane a with a definite choice of Chan-Paton phase (say $\alpha = +1$), while the two lower rows correspond to the opposite choice ($\alpha = -1$). These four D8-branes are then the constituents of a bulk coisotropic D8-brane.

image of Π_2^F under θ. We similarly find that the action of θ on a fractional coisotropic D8-brane is given by

$$
\begin{aligned}
[\Pi_5] &\overset{\theta}{\mapsto} -[\Pi_5] \\
\tfrac{1}{2}F^{\mathrm{b}} + \tfrac{1}{2}F^{\mathrm{tw}} &\overset{\theta}{\mapsto} -\tfrac{1}{2}F^{\mathrm{b}} + \tfrac{\eta}{2}F^{\mathrm{tw}}
\end{aligned}
\tag{3.30}
$$

where we have chosen F^{b} as in (3.18). The choice $\eta = 1$ then implies that, while the untwisted D6-brane charge is left invariant under the action of θ, the twisted D6-brane charge is flipped by a minus sign, and so the total twisted D6-brane charge vanishes (see table 2). This is indeed what we would expect, since for this choice of discrete torsion the RR twisted fields coupling to an A-brane are projected out, and hence such D-brane should not carry any twisted charge.

We can now understand a bulk D8-brane in term of its fractional components, as shown in table 2. Indeed, in order to construct a bulk coisotropic D8-brane we need to consider a fractional D8-brane a with some choice of Chan-Paton phase α and a second D8-brane $\tilde{a}$ with the opposite choice of α. We then add their images under θ and obtain a coisotropic D8-brane in the regular representation of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold group. Notice that each of the pairs $(a, \theta a)$ and $(\tilde{a}, \theta\tilde{a})$ has vanishing D8, D4 and twisted D6-brane charge. In addition, each pair generates a $\mathrm{U}(1)$ vector multiplet, and there are no gauge bosons arising from open strings in mixed sectors like $a\tilde{a}$. Hence the total gauge group of a coisotropic D8-brane in a regular representation is given by $\mathrm{U}(1) \times \mathrm{U}(1)$. In particular, this means that when a bulk D8-brane approaches an orbifold singularity its gauge group gets enhanced as

$$
\mathrm{U}(1) \longrightarrow \mathrm{U}(1) \times \mathrm{U}(1)
\tag{3.31}
$$

in contrast to the enhancement $\mathrm{U}(1) \to \mathrm{U}(2)$ that occurs for D6-branes [26]. As seen above, each of the $\mathrm{U}(1)$ factors in (3.31) can be seen as an independent D8-brane whose net RR charges are those of a fractional D6-brane. More precisely, the net RR charge of a pair $(a, \theta a)$ is given by $[\Pi_3^{F^{\mathrm{b}}}]$, which is specified by the six integer numbers (3.18) of the previous $\mathbf{T}^6$ case. These fractional D8-branes will be the building blocks that we will consider in order to construct $\mathcal{N} = 1$ chiral models.

In fact, F^{b} will be the only quantity to consider when constructing a fractional BPS D8-brane in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, rather than the full F^{fr}. This can be seen from the fact that F^{b} is

the only worldvolume flux present for a generic bulk D8-brane. Taking such bulk D8-brane to an orbifold singularity and splitting it as in table 2 should not affect its BPSness. Thus, if a bulk D8-brane is BPS for some choice of closed string moduli so will be its fractional components. These fractional components inherit the same F^{b} as the bulk D8-brane, but now have a new contribution to F coming from F^{tw}. By construction, the latter should not be involved in the supersymmetry equations (2.4).

A more direct way to reach the same conclusion is to follow the standard procedure to construct D-branes on orbifolds [25]. According to such prescription, the BPSness of a fractional D-brane on $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ is specified by its BPSness in the covering space $\mathbf{T}^6$, with the only condition that the supersymmetry preserved by such D-brane on $\mathbf{T}^6$ needs to be compatible with the bulk supersymmetry preserved by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ action. Again, if a magnetized D8-brane is BPS or not in $\mathbf{T}^6$ only depends on Π_5 and in F^{b}, and so the precise form of F^{tw} will not matter when constructing a BPS D8-brane.[7]

To summarize, we should only concentrate on Π_5 and F^{b} when applying the D-flatness and F-flatness conditions for a fractional coisotropic D8-brane. Because both quantities are specified by the toroidal quanta (3.18), we will obtain similar constraints on the untwisted Kähler and complex structure moduli as those found for the $\mathbf{T}^6$ case. In fact we will see that, for most purposes, we can treat a fractional D8-brane on $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ as a D8-brane on $\mathbf{T}^6$.

Similarly, Π_5 and F^{b} will be the only quantities to consider when checking the consistency of a coisotropic D8-brane model. More precisely, in the present $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold background the zero modes of the RR C_9 and C_5 forms are projected out, and a coisotropic D8-brane may only contribute to C_7 RR divergences via its induced untwisted D6-brane charge $[\Pi_3^{F^{\mathrm{b}}}]$. If in addition we mod out our theory by the orientifold action $\Omega \mathcal{R}(-1)^{F_L}$ specified by (3.22), the only surviving components of C_7 will be those of the form $(C_7)_{x^1 x^2 x^3}$ and $(C_7)_{x^i y^j y^k}$, yielding four independent RR tadpole conditions.

In order to write down such conditions, let us consider a configuration made up of fractional D6 and coisotropic D8-branes, with topological data

$$
\begin{aligned}
\mathrm{D6}_i &: N_i \, (n_i^1, m_i^1)_1 \times (n_i^2, m_i^2)_2 \times (n_i^3, m_i^3)_3 \\
\mathrm{D8}_a &: N_a \, (n_a^1, m_a^1)_1 \times (n_a^{xx}, n_a^{xy}, n_a^{yx}, n_a^{yy})_{23} \\
\mathrm{D8}_b &: N_b \, (n_b^2, m_b^2)_2 \times (n_b^{xx}, n_b^{xy}, n_b^{yx}, n_b^{yy})_{31} \\
\mathrm{D8}_c &: N_c \, (n_c^3, m_c^3)_3 \times (n_c^{xx}, n_c^{xy}, n_c^{yx}, n_c^{yy})_{12}
\end{aligned}
\tag{3.32}
$$

where, following the conventions in [4], the number of D-branes is normalized such that N_α is an even number yielding a $\mathrm{U}(N_\alpha/2)$ gauge group. Cancellation of RR tadpoles then reads

$$
\sum_{i,a,b,c} N_i n_i^1 n_i^2 n_i^3 - N_a n_a^1 n_a^{yy} - N_b n_b^2 n_b^{yy} - N_c n_c^3 n_c^{yy} = 16
$$

---

[7]In the usual orbifold language, the twisted charges F^{tw} do not appear as such, but are encoded in the matrices γ_θ, γ_ω acting on the Chan-Paton degrees of freedom. In this paper we are using a geometric approach which does not make use of the explicit form of γ_θ and γ_ω, but it would be interesting to rederive our results from this more standard procedure.

$$\sum_{i,a,b,c} N_i n_i^1 m_i^2 m_i^3 - N_a n_a^1 n_a^{xx} + N_b m_b^2 n_b^{xy} + N_c m_c^3 n_c^{yx} = -16$$

$$\sum_{i,a,b,c} N_i m_i^1 n_i^2 m_i^3 + N_a m_a^1 n_a^{yx} - N_b n_b^2 n_b^{xx} + N_c m_c^3 n_c^{xy} = -16 \tag{3.33}$$

$$\sum_{i,a,b,c} N_i m_i^1 m_i^2 n_i^3 + N_a m_a^1 n_a^{xy} + N_b m_b^2 n_b^{yx} - N_c n_c^3 n_c^{xx} = -16$$

where we have also included the contribution of the O6-planes introduced by the orientifold quotient.

In addition, in this class of orientifold backgrounds there may be extra consistency conditions which are invisible to one-loop divergences in the worldsheet. Such extra constraints take into account the cancellation of those D-brane charges which cannot be detected by supergravity, but only by a K-theory computation or by using D-brane probes [30]. For the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold at hand and for D6-branes at angles, these extra constraints were computed in [5]. One can repeat the same D-brane probe argument to include D8-branes, obtaining

$$\sum_{i,a,b,c} N_i m_i^1 m_i^2 m_i^3 - N_a m_a^1 n_a^{xx} - N_b m_b^2 n_b^{xx} - N_c m_c^3 n_c^{xx} = 0 \bmod 4$$

$$\sum_{i,a,b,c} N_i m_i^1 n_i^2 n_i^3 - N_a m_a^1 n_a^{yy} + N_b n_b^2 n_b^{yx} + N_c n_c^3 n_c^{xy} = 0 \bmod 4$$

$$\sum_{i,a,b,c} N_i n_i^1 m_i^2 n_i^3 + N_a n_a^1 n_a^{xy} - N_b m_b^2 n_b^{yy} + N_c n_c^3 n_c^{yx} = 0 \bmod 4 \tag{3.34}$$

$$\sum_{i,a,b,c} N_i n_i^1 n_i^2 m_i^3 + N_a n_a^1 n_a^{yx} + N_b n_b^2 n_b^{xy} - N_c m_c^3 n_c^y = 0 \bmod 4$$

which imply the cancellation of K-theoretical $\mathbb{Z}_2$ charges that both D6 and D8-branes may carry. Finally, the CFT analysis of [31] suggests that certain coisotropic D8-branes could carry additional $\mathbb{Z}_2$ charges, again invisible to supergravity. As stated in [31], it is not clear if such additional $\mathbb{Z}_2$ charges could give rise to extra consistency constraints or if (3.33) and (3.34) gather all the consistency conditions of our compactification. For simplicity, we will consider the latter to be the case and leave the analysis of the full set of torsional K-theory charges for future work.

4. Coisotropic branes and chirality

One interesting property of coisotropic D8-branes is that, just like intersecting D6-branes, they give rise to chiral fermions upon compactification. Recall that, in the intersecting D6-brane framework, chiral fermions arise at the intersection locus of the 3-cycles $\Pi_3^{D6_a}$ and $\Pi_3^{D6_b}$ that two D6-branes wrap. These intersections are points in the internal manifold $\mathcal{M}_6$, and they naturally give rise to $D = 4$ fermions in the effective theory [32]. The multiplicity of chiral fermions is given by the signed number of intersections

$$I_{ab} = [\Pi_3^{D6_a}] \circ [\Pi_3^{D6_b}] \tag{4.1}$$

which only depends on the D6-brane homological charges $[\Pi_3^{D6_a}]$ and $[\Pi_3^{D6_b}]$.

If we now consider two D8-branes D8$_a$ and D8$_b$ they may also intersect, but they would do so in a 4-cycle $\mathcal{S}_4$ and there is a priori no reason to expect $D = 4$ chirality to arise from such intersection. However, because coisotropic D8-branes carry a worldvolume flux F, there will be a relative flux $F_{ab} = F_b - F_a$ coupling to the open strings in the D8$_a$D8$_b$ sector. This flux will modify the internal Dirac index of the $D = 8$ theory compactified on $\mathcal{S}_4$ and, in particular, it will be a source of $D = 4$ chirality via the coupling $\int_{\mathcal{S}_4} F_{ab}^2$. The same story will apply to a D6$_a$-D8$_b$ system. Now the intersection locus is given by a 2-cycle $\mathcal{S}_2$, and a natural source of $D = 4$ chirality will be given by the quantity $\int_{\mathcal{S}_2} F_{ab}$.

We thus see that, when including D8-branes into the picture, the source of chirality in type IIA compactifications is not only given by the intersection of D-branes, but rather by a combination of intersection and magnetization mechanism. This reminds of the case of type IIB theory, where such combination also arises from compactification with D5 or D7-branes. From the type IIB literature we know that computing the chiral spectrum in those cases may be quite involved, and that it requires describing our D-branes as coherent sheaves and computing the so-called 'Ext' groups among them [33]. In principle we could apply a similar procedure to compute the matter content of our D8$_a$-D8$_b$ and D6$_a$-D8$_b$ system. However, we will see below that for coisotropic D8-branes on $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ the number of chiral fermions can be expressed in a rather simple way. Indeed, let $[\Pi_3^{D8}]$ be the D6-brane charge induced on a coisotropic D8-brane. Then the net chirality on a D8$_a$-D8$_b$ and D6$_a$-D8$_b$ system is given by

$$I_{ab} = [\Pi_3^{D8_a}] \circ [\Pi_3^{D8_b}] \quad \text{and} \quad I_{ab} = [\Pi_3^{D6_a}] \circ [\Pi_3^{D8_b}] \tag{4.2}$$

respectively. In fact, we would expect such expressions to be valid for general Calabi-Yau manifold $\mathcal{M}_6$. Notice that when $[\Pi_3^{D8}]$ is trivial in the homology of $\mathcal{M}_6$ then any of these intersection numbers will vanish, but then our D8-brane could never be a BPS object. Hence the source of stability for a coisotropic D8-brane (i.e., the worldvolume flux F) turns out to be also the source of $D = 4$ chirality.

4.1 D6-D6 chirality

Let us first review the case where chirality arises from two intersecting D6-branes. The simplest case is that of $\mathbf{T}^6 = \mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$ and two D6-branes at angles

$$\begin{aligned} \text{D6}_a &: N_a \ (n_a^1, m_a^1)_1 \times (n_a^2, m_a^2)_2 \times (n_a^3, m_a^3)_3 \\ \text{D6}_b &: N_b \ (n_b^1, m_b^1)_1 \times (n_b^2, m_b^2)_2 \times (n_b^3, m_b^3)_3 \end{aligned} \tag{4.3}$$

yielding a gauge group $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$. The number of chiral fermions in the bifundamental representation $(N_a, \overline{N_b})$ is given by the intersection number (4.1), which is computed by taking the Poincaré dual 3-forms

$$\begin{aligned} \omega_a &= -(n_a^1 dy^1 - m_a^1 dx^1) \wedge (n_a^2 dy^2 - m_a^2 dx^2) \wedge (n_a^3 dy^3 - m_a^3 dx^3) \\ \omega_b &= -(n_b^1 dy^1 - m_b^1 dx^1) \wedge (n_b^2 dy^2 - m_b^2 dx^2) \wedge (n_b^3 dy^3 - m_b^3 dx^3) \end{aligned} \tag{4.4}$$

and integrating them over $\mathbf{T}^6$ to yield

$$I_{ab} = \int_{\mathbf{T}^6} \omega_a \wedge \omega_b, = (n_a^1 m_b^1 - n_b^1 m_a^1) \cdot (n_a^2 m_b^2 - n_b^2 m_a^2) \cdot (n_a^3 m_b^3 - n_b^3 m_a^3) \tag{4.5}$$
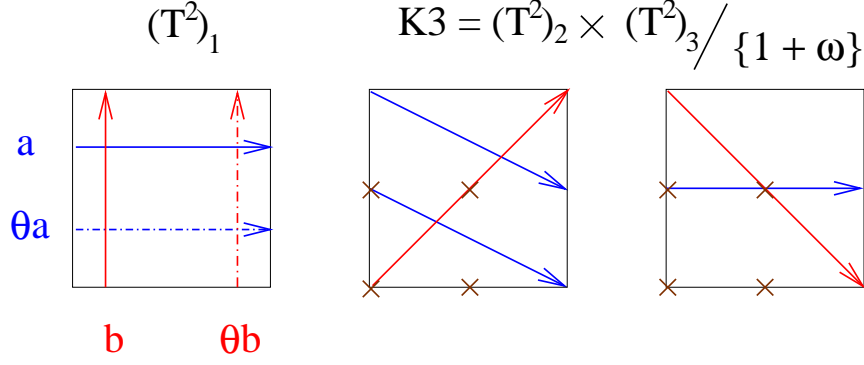
Figure 3: Chirality from a D6-D6 system on $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$.

When considering the orbifold $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ the situation is more involved but, as shown in [4] by means of CFT arguments, the number of chiral fermions is again given by (4.5), and the only difference is that the gauge group is now $\mathrm{U}(N_a/2) \times \mathrm{U}(N_b/2)$. Let us rederive this result by the more geometric approach followed in [28]. Since N_a is an even number, we can separate a stack of $N_a/2$ $\mathrm{D6}_a$-branes away from the fixed points on $(\mathbf{T}^2)_1$, the remaining $N_a/2$ $\mathrm{D6}_a$-branes being their images under θ. We can repeat the same procedure with $N_b/2$ $\mathrm{D6}_b$-branes, without changing the gauge group or chiral spectrum of our configuration [34] (see figure 3). Just like the fractional D8-brane discussed above, these D6-branes see the geometry $(\mathbf{T}^2)_1 \times \mathbf{T}^4/\mathbb{Z}_2^\omega$, so they wrap a 3-cycle of the form

$$
\begin{aligned}
\mathrm{D6}_\alpha &: N_\alpha/2 \ (n_\alpha^1, m_\alpha^1)_1 \times \left(\tfrac{1}{2}\Pi_{2,\alpha}^{\mathrm{b}} + \tfrac{1}{2}\Pi_{2,\alpha}^{\mathrm{tw}}\right) \\
\mathrm{D6}_{\theta\alpha} &: N_\alpha/2 \ (n_\alpha^1, m_\alpha^1)_1 \times \left(\tfrac{1}{2}\Pi_{2,\alpha}^{\mathrm{b}} - \tfrac{1}{2}\Pi_{2,\alpha}^{\mathrm{tw}}\right) \\
\Pi_{2,\alpha}^{\mathrm{b}} &= (n_\alpha^2, m_\alpha^2)_2 \times (n_\alpha^3, m_\alpha^3)_3
\end{aligned}
\tag{4.6}
$$

where $\alpha = a, b$ and we have decomposed a minimal 2-cycle of $\mathbf{T}^4/\mathbb{Z}_2$ as in (3.27). Again, the relative sign in $\Pi_{2,\alpha}^{\mathrm{tw}}$ follows from the specific choice of discrete torsion $\eta = 1$ in (3.29).

In order to compute the chiral spectrum we sum over the intersections of $\mathrm{D6}_a\mathrm{D6}_b$ and $\mathrm{D6}_a\mathrm{D6}_{\theta b}$, the other two sectors being mapped to these by the action of θ. We then find

$$
\begin{aligned}
I_{ab} &= [\Pi_3^{D6_a}] \circ [\Pi_3^{D6_b}] + [\Pi_3^{D6_a}] \circ [\Pi_3^{D6_{\theta b}}] \\
&= \left([(n_a^1, m_a^1)] \circ_{\mathbf{T}^2} [(n_b^1, m_b^1)]\right) \cdot \left(\tfrac{1}{2}[\Pi_{2,a}^{\mathrm{b}}] \circ_{\mathbf{K3}} [\Pi_{2,b}^{\mathrm{b}}]\right) \\
&= \left(-(n_a^1 m_b^1 - n_b^1 m_a^1)\right) \cdot \left(-(n_a^2 m_b^2 - n_b^2 m_a^2) \cdot (n_a^3 m_b^3 - n_b^3 m_a^3)\right)
\end{aligned}
\tag{4.7}
$$

where we have used the fact that $[\Pi_{2,\alpha}^{\mathrm{b}}] \circ_{\mathbf{K3}} [\Pi_{2,\alpha}^{\mathrm{tw}}] = 0$ and that the embedding of $H_2(\mathbf{T}^4, \mathbb{Z})$ into $H_2(\mathbf{K3}, \mathbb{Z})$ involves a factor of two [28].[8] Finally, one can check that both summands in the first line of (4.7) have the same sign, and so there is no vector-like pairs arising from the D6-D6 system.

4.2 D6-D8 chirality

Let us now consider a $\mathrm{D6}_a$-$\mathrm{D8}_b$ system on $\mathbf{T}^6$. In particular, let us take the case where the

---

[8]Recall that in our conventions $\int_{(\mathbf{T}^2)_i} dy^i \wedge dx^i = 1$ (see footnote 4), and so there is a relative sign with respect to the usual intersection product used for $\mathbf{T}^2$.
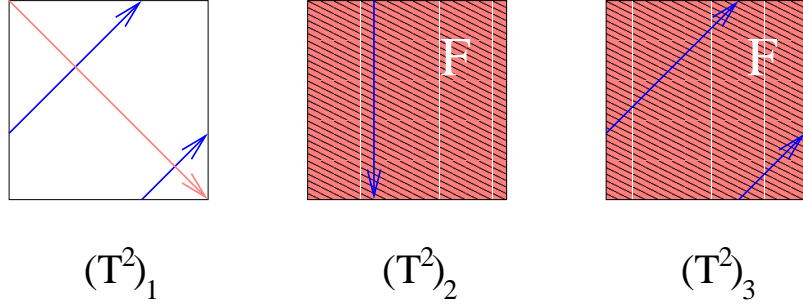
Figure 4: Chirality from a D6-D8 system on $\mathbf{T}^6$.

direction not filled by the D8-brane is on $(\mathbf{T}^2)_i$, so that we have

$$
\begin{aligned}
\text{D6}_a &: N_a \ (n_a^1, m_a^1)_1 \times (n_a^2, m_a^2)_2 \times (n_a^3, m_a^3)_3 \\
\text{D8}_b &: N_b \ (n_b^i, m_b^i)_i \times (n_b^{xx}, n_b^{xy}, n_b^{yx}, n_b^{yy})_{jk}
\end{aligned}
\tag{4.8}
$$

as illustrated in figure 4, with $i = 1$. Both D-branes intersect $|n_a^i m_b^i - n_b^i m_a^i|$ times on $(\mathbf{T}^2)_i$, and each intersection is a $\mathbf{T}^2$ of the form $\Pi_2^{D6_a} = \{(n_a^j, m_a^j)_j \times (n_a^k, m_a^k)_k\}$. Thus, each intersection can be seen as a $D = 6$ theory compactified on $\mathbf{T}^2$, with a $D = 6$ chiral fermion arising from the D6$_a$D8$_b$ sector. Such $D = 6$ theory is deformed by the presence of a relative flux F_{ab}, which is nothing but the pull-back of F^{D8_b} on $\Pi_2^{D6_a}$. The multiplicity of chiral fermions associated to the Landau levels of this toron system is given by the index

$$
\int_{\Pi_2^{D6_a}} F_b = \int_{\mathbf{T}^4} \tilde{\omega}_a \wedge F_b = [\Pi_2^{D6_a}] \circ_{\mathbf{T}^4} [\Pi_2^{D8_b}] = I_{ab}^{\mathbf{T}^4}
\tag{4.9}
$$

where $\tilde{\omega}_a$ is the Poincaré dual of $\Pi_2^{D6_a}$ on $\mathbf{T}^4$. If we multiply this spectrum by the number of intersections on $(\mathbf{T}^2)_i$ we arrive to the total multiplicity

$$
I_{ab} = I_{ab}^{(\mathbf{T}^2)_i} \cdot I_{ab}^{(\mathbf{T}^4)_{jk}} = [\Pi_3^{D6_a}] \circ [\Pi_3^{D8_b}]
\tag{4.10}
$$

as advanced in (4.2). In terms of the integer numbers (4.8) we have a total of

$$
I_{ab} = -(n_a^i m_b^i - n_b^i m_a^i) \cdot (n_a^j n_a^k n_b^{xx} + n_a^j m_a^k n_b^{xy} + m_a^j n_a^k n_b^{yx} + m_a^j m_a^k n_b^{yy})
\tag{4.11}
$$

chiral multiplets transforming in the bifundamental representation $(N_a, \overline{N_b})$.

In order to compute the chiral spectrum of the D6$_a$-D8$_b$ system in the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold we can follow the same strategy used for intersecting D6-branes. We separate both D6 and D8-branes away from the $\mathbb{Z}_2 \times \mathbb{Z}_2$ fixed points on one of the two-tori, say $(\mathbf{T}^2)_1$, and then we describe them as fractional branes on $\mathbf{T}^2 \times \mathbf{T}^4/\mathbb{Z}_2$. The main difference with the computation of $I_{ab}^{\mathbf{T}^4}$ is that now $\Pi_2^{D6_a}$ and F_b have each a component from the twisted sectors of **K3**. However, the twisted contributions to $I_{ab}^{\mathbf{K3}}$ will cancel once that we have summed over the sectors D8$_b$ and D8$_{\theta b}$, and so we will recover (4.11). The only difference with $\mathbf{T}^6$ will be that now the gauge group is given by $\mathrm{U}(N_a/2) \times \mathrm{U}(N_b/2)$, and so (4.11) indicates the number of chiral multiplets in the representation $(N_a/2, \overline{N_b}/2)$.
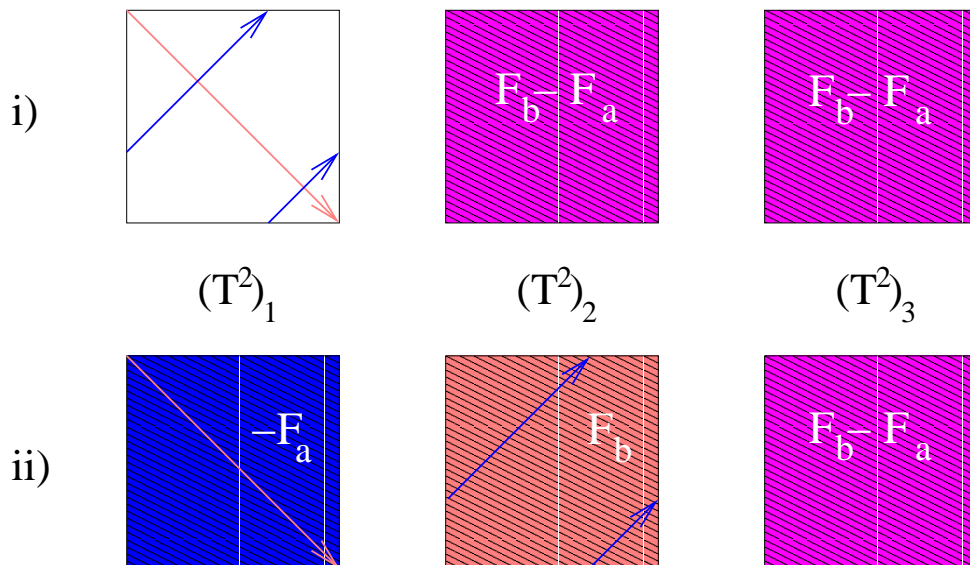
Figure 5: Chirality from a D8-D8 system on $\mathbf{T}^6$.

4.3 D8-D8 chirality

Finally, let us consider a D8$_a$-D8$_b$ system. On $\mathbf{T}^6$ our coisotropic D8-branes are described by

$$
\begin{aligned}
\text{D8}_a &: N_a \ (n_a^i, m_a^i)_i \times (n_a^{xx}, n_a^{xy}, n_a^{yx}, n_a^{yy})_{jk} \\
\text{D8}_b &: N_b \ (n_b^{i'}, m_b^{i'})_{i'} \times (n_b^{xx}, n_b^{xy}, n_b^{yx}, n_b^{yy})_{j'k'}
\end{aligned}
\tag{4.12}
$$

where $\{ijk\}$ and $\{i'j'k'\}$ are both even permutations of $\{123\}$. Although D8$_a$ and D8$_b$ will always intersect on a $\mathbf{T}^4$, we can distinguish between two different cases:

- $i = i'$ [figure 5, i)]

  In this case D8$_a$ and D8$_b$ intersect $|n_a^i m_b^i - n_b^i m_a^i|$ on $(\mathbf{T}^2)_i$, and they overlap on $(\mathbf{T}^2)_j \times (\mathbf{T}^2)_k$. On each intersection we have a $D = 8$ gauge theory compactified on $\mathbf{T}^4$, with a relative flux $F_{ab} = F_b - F_a$ acting on the D8$_a$D8$_b$ sector. The multiplicity associated to the Landau levels of this compactification is now given by
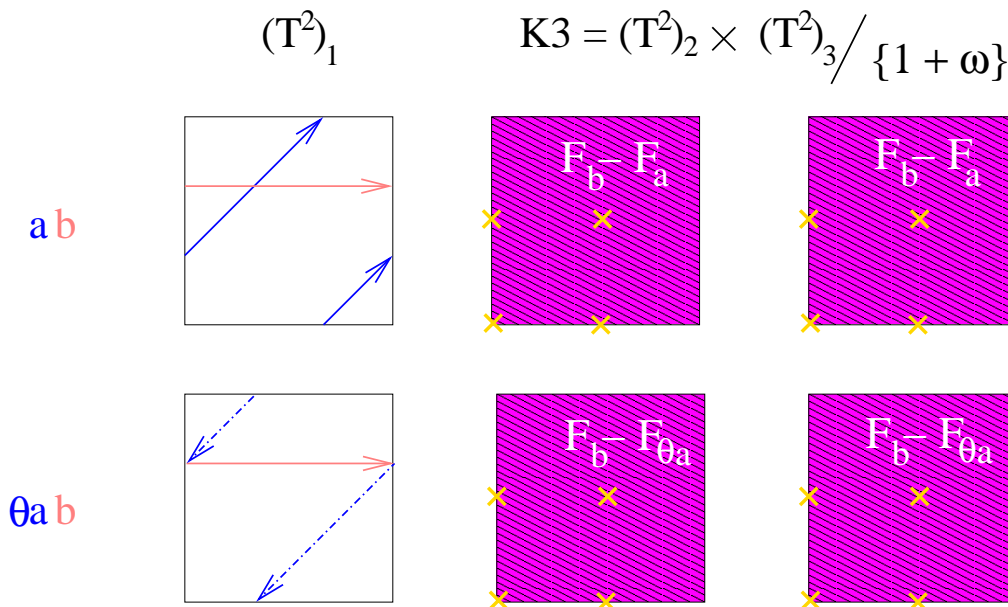
  $$
  -\frac{1}{2} \int_{(\mathbf{T}^2)_j \times (\mathbf{T}^2)_k} (F_b - F_a)^2 = (n_b^{xx} - n_a^{xx})(n_b^{yy} - n_a^{yy}) - (n_b^{xy} - n_a^{xy})(n_b^{yx} - n_a^{yx}) \tag{4.13}
  $$

  and so the total number of chiral fermions is

  $$
  -(n_a^i m_b^i - n_b^i m_a^i) \cdot \left((n_b^{xx} - n_a^{xx})(n_b^{yy} - n_a^{yy}) - (n_b^{xy} - n_a^{xy})(n_b^{yx} - n_a^{yx}) \right) \tag{4.14}
  $$

- $i \neq i'$ [figure 5, ii)]

  In this case both D8-branes intersect just once and the $\mathbf{T}^4$ where they overlap is of the form $(\mathbf{T}^4)_{ab} = (n_a^i, m_a^i) \times (n_b^{i'}, m_b^{i'}) \times (\mathbf{T}^2)_k$, with $i \neq k \neq i'$. The multiplicity of

Figure 6: Chirality from a D8-D8 system on $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$.

the $\mathrm{D8}_a\mathrm{D8}_b$ is now given by the integral

$$
\begin{aligned}
-\int_{(\mathbf{T}^4)_{ab}} \tfrac{1}{2}(F_b - F_a)^2 &= \int_{(n_a^i, m_a^i) \times (n_b^{i'}, m_b^{i'}) \times (\mathbf{T}^2)_k} F_a \wedge F_b \\
&= \int_{\mathbf{T}^6} (n_a^i dy^i - m_a^i dx^i) \wedge F_a \wedge (n_b^{i'} dy^{i'} - m_b^{i'} dx^{i'}) \wedge F_b \quad (4.15) \\
&= \int_{\mathbf{T}^6} \omega_a \wedge \omega_b = [\Pi_3^{D8_a}] \circ [\Pi_3^{D8_b}]
\end{aligned}
$$

where $[\omega_\alpha]$ is the Poincaré dual of $[\Pi_3^{D8_\alpha}]$. In terms of the D8-brane quanta (4.12) we have, for $i' = j$,

$$
\begin{aligned}
I_{ab} = {}& n_a^i m_b^{i'}(n_a^{yy} n_b^{xx} - n_a^{yx} n_b^{yx}) - m_a^i n_b^{i'}(n_a^{xx} n_b^{yy} - n_a^{xy} n_b^{xy}) \\
& - n_a^i n_b^{i'}(n_a^{xx} n_b^{yx} - n_a^{xy} n_b^{xx}) - m_a^i m_b^{i'}(n_a^{yy} n_b^{xy} - n_a^{yx} n_b^{yy})
\end{aligned} \quad (4.16)
$$

and for $i' = k$ we just need to interchange $(i \leftrightarrow i')$ and $(a \leftrightarrow b)$ in (4.16) in order to compute $-I_{ab}$.

Notice that (4.16) can be put in the form (4.2), but that this is not the case for (4.14). This is because $\mathbf{T}^6$ contains non-trivial 5 and 1-cycles, and so there are non-trivial RR charges associated to both D8 and D4-branes wrapped on such cycles. One would expect such charges to play a role when computing the chiral spectrum of a compactification, in the same sense that a D4-D8 system gives rise to $D = 4$ chirality when both D-branes have a non-trivial intersection. In Calabi-Yau manifolds with $b_1 = 0$ the only conserved charges should be related to 3-cycles Π_3 and then we should be able to compute the chiral spectrum of a D8-D8 system by means of (4.2).

This is indeed the case of $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, which we now analyze. We again describe our fractional D8-branes in terms of the covering space $\mathbf{T}^2 \times \mathbf{T}^4/\mathbb{Z}_2$, as shown in figure 2.

Recall that now F_a and F_b are 2-forms in **K3** and that they are mapped to (3.30) by the action of θ, so that we have

$$
\begin{aligned}
\text{D8}_\alpha &: \; N_\alpha/2 \; (n_\alpha^i, m_\alpha^i)_i \times \mathbf{K3}, \quad F_\alpha^{\text{fr}} = \tfrac{1}{2}F_\alpha^{\text{b}} + \tfrac{1}{2}F_\alpha^{\text{tw}} \\
\text{D8}_{\theta\alpha} &: \; N_\alpha/2 \; (-n_\alpha^i, -m_\alpha^i)_i \times \mathbf{K3}, \quad F_{\theta\alpha}^{\text{fr}} = -\tfrac{1}{2}F_\alpha^{\text{b}} + \tfrac{1}{2}F_\alpha^{\text{tw}}
\end{aligned}
\tag{4.17}
$$

$$
F_\alpha^{\text{b}} = n_\alpha^{xx}\, dx^j \wedge dx^k + n_\alpha^{xy}\, dx^j \wedge dy^k + n_\alpha^{yx}\, dy^j \wedge dx^k + n_\alpha^{yy}\, dy^j \wedge dy^k
$$

Adding up the contributions from the D8_a-D8_b and $\text{D8}_{\theta a}$-D8_b sectors of figure 6 we arrive to a chiral spectrum given by

$$
\begin{aligned}
I_{ab} &= (n_a^i m_b^i - n_b^i m_a^i) \cdot \left(\int_{\mathbf{K3}} \tfrac{1}{2}(F_b^{\text{fr}} - F_a^{\text{fr}})^2 - \int_{\mathbf{K3}} \tfrac{1}{2}(F_b^{\text{fr}} - F_{\theta a}^{\text{fr}})^2 \right) \\
&= -(n_a^i m_b^i - n_b^i m_a^i) \cdot \int_{\mathbf{K3}} F_b^{\text{fr}} \wedge \left(F_a^{\text{fr}} - F_{\theta a}^{\text{fr}} \right) \\
&= -(n_a^i m_b^i - n_b^i m_a^i) \cdot \tfrac{1}{2} \int_{\mathbf{K3}} F_b^{\text{b}} \wedge F_a^{\text{b}} = [\Pi_3^{D8_a}] \circ [\Pi_3^{D8_b}]
\end{aligned}
\tag{4.18}
$$

which can indeed be expressed as the intersection product of two 3-cycles. As usual, this quantity only depends on the bulk wrapping and magnetic numbers (4.12), in terms of which it reads

$$
I_{ab} = (n_a^i m_b^i - n_b^i m_a^i) \cdot (n_a^{xx} n_b^{yy} + n_a^{yy} n_b^{xx} - n_a^{xy} n_b^{yx} - n_a^{yx} n_b^{xy})
\tag{4.19}
$$

Finally, the same procedure can be applied to the D8-D8 system of figure 5, ii) in order to compute its chiral spectrum when embedded in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$. In this case the computation parallels those of the D6-D6 and D6-D8 systems, in the sense that the contributions coming from twisted sectors cancel each other and we end up with the intersection number I_{ab} again given by (4.16).

4.4 Summary

Let us summarize the above results on the chiral spectrum arising from D6-branes at angles and coisotropic D8-branes. Although now there are several possibilities in order to produce chirality from a pair of D-branes, finding the chiral spectrum of a compactification always boils down to compute the intersection number I_{ab} between two homology classes of 3-cycles $[\Pi_3^a]$ and $[\Pi_3^b]$. As advanced in (4.1) and (4.2), such 3-cycles may correspond to those wrapped by a D6-brane $[\Pi_3^{D6}]$ or to the D6-brane charge which is dissolved into a coisotropic D8-brane $[\Pi_3^{D8}]$.

Because of the choice of discrete torsion taken on the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ background, neither the D6-branes nor the D8-branes carry any twisted charge, and so the homological charges $[\Pi_3^{D6}]$ and $[\Pi_3^{D8}]$ can be understood in terms of (a subsector of) the $\mathbf{T}^6$ cohomology. This implies that the intersection numbers I_{ab} can be expressed in terms of the wrapping and magnetic numbers (3.32) that define a D6-brane at angles or a coisotropic D8-brane in $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$. In particular, they take the expressions (4.5), (4.11), (4.16) or (4.19) depending on which particular pair of D-branes we are considering.

Once these intersection numbers have been computed, everything works as in the case of D6-branes at angles discussed in [4]. In particular, because we are performing the same orientifold projection that introduces O6-planes in our theory, we need to consider the D6's and D8's orientifold images. In the case of the D8-branes, this means that we need to add

Sector	Representation
aa	$U(N_a/2)$ vector multiplet
$ab + ba$	I_{ab} $(\square_a, \overline{\square}_b)$ chiral multiplets
$ab' + b'a$	$I_{ab'}$ $(\square_a, \square_b)$ chiral multiplets
$aa' + a'a$	$\frac{1}{2}(I_{aa'} - 4I_{aO})$ $\square\square$ chiral multiplets
	$\frac{1}{2}(I_{aa'} + 4I_{aO})$ $\begin{array}{c}\square\\\square\end{array}$ chiral multiplets

Table 3: Gauge groups and chiral spectrum spectrum for $\mathcal{N} = 1$ intersecting D6-branes and coisotropic D8-branes in the $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold considered in this paper.

the images (3.23) in our compactification and that we must also compute the intersection numbers $I_{ab'}$. Finally, the D6$_a$-D6$_{a'}$ and D8$_a$-D8$_{a'}$ sectors may give rise to symmetric and/or antisymmetric chiral matter, and this part of the spectrum also depends on the intersection number I_{aO} between $[\Pi_3^{D6}]$, $[\Pi_3^{D8}]$ and the O6-plane homology class, given by

$$[\Pi_3^{O6}] = [(1,0)\,(1,0)\,(1,0)] + [(1,0)\,(0,1)\,(0,-1)]$$
$$+ [(0,1)\,(1,0)\,(0,-1)] + [(0,1)\,(0,-1)\,(1,0)]. \qquad (4.20)$$

Such chiral spectrum is then summarized by table 3.

5. Effective field theory

Having constructed BPS coisotropic D8-branes in $\mathbf{T}^6$ and in the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold, we would now like to understand what is the effective field theory associated to them. In particular, we would like to derive the expressions for the F and D-terms that appear when the supersymmetry conditions (2.8) are no longer met. As usual, these quantities can be extracted from the scalar potential generated in the $D = 4$ effective theory and, more precisely, from the contribution of the D8-brane DBI action to such scalar potential.

We have already encountered an effective field theory quantity in the previous section, namely the spectrum of $D = 4$ chiral fermions arising from a D8-brane when intersecting others. In that case we found that everything depends on the D6-brane induced charge on the D8-brane, so one may wonder if this also true for any other quantity. We will see that this is indeed the case for some elements of the effective field theory, like the gauge kinetic functions, Fayet-Iliopoulos terms and massive U(1)'s of a D8-brane and that, in these cases, everything works like for a D6-brane wrapping a non-factorizable 3-cycle. On the other hand, there will be important differences when analyzing the $D = 4$ superpotential. In particular, we will see that a open-closed superpotential will couple the D8-brane moduli to the Kähler moduli of the compactification, and that the generation of Yukawa couplings may or may not involve world-sheet instantons.

Because our coisotropic D8-branes do not carry any RR twisted charge and we are working in the orbifold limit, all the effective theory quantities will depend on the toroidal

D8-branes charges (3.18) and on the closed untwisted moduli of the compactification. Recall that in the IIA orientifold that we are considering there are three untwisted Kähler moduli T_i, defined in terms of the complexified Kähler form J_c as shown in (3.3).[9] The remaining moduli are the dilaton S and the three complex structure moduli U_i, which can be encoded in the complexified 3-form [35]

$$(4\pi^2\alpha')^{3/2}\,\Omega_c = C_3 + i\mathrm{Re}\,(C\Omega) \tag{5.1}$$

where in our conventions $C = e^{-\phi}$. The RR 3-form C_3 is to be expanded in a basis of 3-forms invariant under the orientifold action. In our case,

$$\Omega_c = iS\,dx^1 \wedge dx^2 \wedge dx^3 - i\sum_{i \neq j \neq k} U_i\,dx^i \wedge dy^j \wedge dy^k\ . \tag{5.2}$$

It then follows that

$$\mathrm{Re}\,S = e^{-\phi}R_1R_2R_3 \quad;\quad \mathrm{Re}\,U_i = e^{-\phi}R_iR_jR_k\tau_j\tau_k \quad,i \neq j \neq k\ . \tag{5.3}$$

For future purposes we introduce the four-dimensional dilaton given by

$$e^{4\phi_4} = \frac{e^{4\phi}}{\mathcal{V}^2} = (\mathrm{Re}\,S\,\mathrm{Re}\,U_1\,\mathrm{Re}\,U_2\,\mathrm{Re}\,U_3)^{-1}\ , \tag{5.4}$$

where $\mathcal{V} = R_1^2R_2^2R_3^2\tau_1\tau_2\tau_3$ is the volume of the internal $\mathbf{T}^6$ in string units.

The effective action for a Dp-brane is the sum of two terms. The first is the Dirac-Born-Infeld action given by

$$S_{\mathrm{DBI}} = -\mu_p \int_{\Sigma_{p+1}} d^{p+1}\xi\, e^{-\phi}\,\mathrm{Tr}\,\sqrt{\det(G + B + 2\pi\alpha'F)}\ , \tag{5.5}$$

where Σ_{p+1} is the D-brane worldvolume. As usual, G and B are the pullbacks of the corresponding $D{=}10$ tensors, whereas F is the worldvolume field strength. The second term is the Chern-Simons action

$$S_{\mathrm{CS}} = -\mu_p \int_{\Sigma_{p+1}} \mathcal{C} \wedge e^{\mathcal{F}}\ , \tag{5.6}$$

where $\mathcal{C}$ is a formal sum of RR forms.

As usual, these quantities simplify for D-branes on tori or toroidal orbifolds. For instance, if we consider a D8$_a$-brane with data given by (3.18), we can derive the useful result,

$$\begin{aligned}
\det\,(G + \mathcal{F})|_{\Pi_5} = (4\pi^2\alpha')^5 R_1^2R_2^2R_3^2\,&\left((n_a^i)^2 + (m_a^i\tau_i)^2\right)[(n_a^{yy} - n_a^{xx}\tau_j\tau_k)^2 + (n_a^{xy}\tau_j + n_a^{yx}\tau_k)^2 \\
&+ \frac{1}{R_j^2R_k^2}\,|T_jT_k - n_a^{xy}n_a^{yx} + n_a^{xx}n_a^{yy}|^2\,]
\end{aligned} \tag{5.7}$$

---

[9]In this section we will express the real part of these Kähler moduli as $\mathrm{Re}\,T_i = R_i^2\tau_i$, where R_i is the compactification radius in the x^i direction (in units of $2\pi\sqrt{\alpha'}$) and τ_i has been defined in (3.2).

where $|_{\Pi_5}$ stands for the pull-back on the 5-cycle that the D8-brane is wrapping. Observe that using the F and D-flatness conditions (3.17) and (3.21) leads to

$$\int_{\Pi_5} \sqrt{\det(G + \mathcal{F})} = \int_{\Pi_5} \mathcal{F} \wedge \operatorname{Re}\Omega = 2\pi\alpha' \int_{\Pi_5} F \wedge \operatorname{Re}\Omega \qquad (5.8)$$

where we have used that the B-field is a (1,1)-form and hence $B \wedge \Omega = 0$. As we will shortly explain, moving away from these supersymmetry conditions allows instead to deduce F and D-terms in the scalar potential.

5.1 Gauge coupling constants

Let us consider a type II compactification, and a gauge theory arising from a Dp-brane wrapping a $(p-3)$-cycle Π_{p-3} on the internal manifold. One basic question is which is the gauge coupling constant of such theory. As it is well known, this follows from reducing the Dirac-Born-Infeld action (5.5) over the compact Π_{p-3}, and reading the gauge coupling from the resulting $D=4$ action. In this way we obtain the general result [36]

$$\frac{2\pi}{g_p^2} = (2\pi)^{3-p} \alpha'^{\frac{3-p}{2}} \int_{\Pi_{p-3}} e^{-\phi} \sqrt{\det(G + \mathcal{F})} \qquad (5.9)$$

In an $\mathcal{N}=1$ supersymmetric theory it must be that $1/g_p^2$ is given by the real part of the holomorphic gauge kinetic function f_p. Furthermore, the imaginary part of f_p determines the axionic couplings. These couplings can be derived from reducing the Chern-Simons action (5.6) over Π_{p-3} and so, eventually, $\operatorname{Im} f_p$ can be also deduced. For instance, for a D6-brane, $\operatorname{Im} f_6 \sim \int_{\Pi_3} C_3$, whereas for a coisotropic D8-brane,

$$\operatorname{Im} f_8 = -\frac{1}{2\pi}(4\pi^2\alpha')^{-3/2} \int_{\Pi_5} C_3 \wedge \frac{F}{2\pi} \ . \qquad (5.10)$$

Clearly, $\operatorname{Im} f_8$ depends linearly on the imaginary part of the complex structure moduli that come from the RR 3-form. On the other hand, $\operatorname{Re} f_8$ seems to have a complicated dependence on the NSNS piece of the moduli. However, once the supersymmetric conditions are used we can easily reconstruct the holomorphic gauge function.

Indeed, substituting eq. (5.8) in the gauge coupling constant gives

$$\operatorname{Re} f_8 = \frac{1}{g_8^2} = \frac{1}{2\pi}(4\pi^2\alpha')^{-3/2} \int_{\Pi_5} \frac{F}{2\pi} \wedge \operatorname{Re}(e^{-\phi}\Omega) \ . \qquad (5.11)$$

so combining with the imaginary part yields

$$f_8 = -\frac{i}{2\pi} \int_{\Pi_5} \frac{F}{2\pi} \wedge \Omega_c = -\frac{i}{2\pi} \int_{[\Pi_3]} \Omega_c \qquad (5.12)$$

where $[\Pi_3]$ is the Poincaré dual of $[F/2\pi]$ in Π_5. It is thus clear that the gauge kinetic function only depends on the D6-brane charge induced on the coisotropic D8-brane.

In particular, in our toroidal compactification we can use (5.2) to express Ω_c in terms of the moduli, so that for the D8$_a$-brane (3.18) we obtain

$$f_{8a} = -n_a^i(n_a^{yy}S - n_a^{xx}U_i) - m_a^i(n_a^{yx}U_j + n_a^{xy}U_k) \qquad (5.13)$$

which explicitly shows that f_{8a} is a holomorphic function of the dilaton and the complex structure moduli. In addition, using the dictionary (3.19) one can check that this expression matches the one obtained in [37] for intersecting D6-branes.

5.2 Scalar potential

Basically, the contribution of a Dp-brane to the scalar potential of an effective theory can be computed from its tension, as derived from the Dirac-Born-Infeld action. For example, for a coisotropic D8-brane we have

$$V_{\text{D8}} = \mu_8 \int_{\Pi_5} e^{4\phi_4} e^{-\phi} \sqrt{\det(G + \mathcal{F})} \, . \tag{5.14}$$

where the factor $e^{4\phi_4}$ appears because the action term is actually $\int d^4x \sqrt{\tilde{g}} \, V_{\text{D8}}$, $\tilde{g}_{\mu\nu}$ being the 4-dimensional metric in Einstein frame. In the toroidal compactification the four-dimensional dilaton is given by (5.4).

To proceed further, we consider a D8$_a$ brane for which the Dirac-Born-Infeld determinant is computed in (5.7). As in [17], it is convenient to define D-term and F-term densities as

$$
\begin{aligned}
e^{-\phi}\text{Re}\left(\mathcal{F} \wedge \Omega\right)|_{\Pi_5} &= (4\pi^2\alpha')^{5/2} \, \Theta_a \, d\text{vol}_{\Pi_5} \, , \\
e^{-\phi}\text{Im}\left(\mathcal{F} \wedge \Omega\right)|_{\Pi_5} &= (4\pi^2\alpha')^{5/2} \, \mathcal{D}_a \, d\text{vol}_{\Pi_5} \, , \\
(\mathcal{F} + iJ)^2|_{\mathbf{T}^4} &= (4\pi^2\alpha')^2 \, \mathcal{Q}_a \, d\text{vol}_{\mathbf{T}^4} \, .
\end{aligned}
\tag{5.15}
$$

These quantities have been basically computed in (3.17) and (3.20). Explicitly,

$$
\begin{aligned}
\Theta_a &= \left[-n_a^i(n_a^{yy}\text{Re}\,S - n_a^{xx}\text{Re}\,U_i) - m_a^i(n_a^{yx}\text{Re}\,U_j + n_a^{xy}\text{Re}\,U_k)\right] \, , \\
e^{2\phi_4}\mathcal{D}_a &= \left[-\frac{m_a^i n_a^{yy}}{\text{Re}\,U_i} + \frac{m_a^i n_a^{xx}}{\text{Re}\,S} + \frac{n_a^i n_a^{xy}}{\text{Re}\,U_j} + \frac{n_a^i n_a^{yx}}{\text{Re}\,U_k}\right] \equiv D_a \, , \\
\mathcal{Q}_a &= \left[T_j T_k - n_a^{xy}n_a^{yx} + n_a^{xx}n_a^{yy}\right] \, ,
\end{aligned}
\tag{5.16}
$$

where we have also used (5.3) and (5.4). Notice that $\Theta_a = \text{Re}\,f_{8a}$.

The next step is to identify the terms in (5.7) to arrive at

$$V_{\text{D8}_a} = \mu_8(4\pi^2\alpha')^{5/2} e^{4\phi_4} \sqrt{\Theta_a^2 + \mathcal{D}_a^2 + e^{-2\phi}||\ell_i||^2|\mathcal{Q}_a|^2} \, , \tag{5.17}$$

where $||\ell_i||^2 = 4\pi^2\alpha' R_i^2 \left((n_a^i)^2 + (m_a^i\tau_i)^2\right)$. Expanding the square root readily gives

$$V_{\text{D8}_a} = \frac{1}{8\pi^3\alpha'^2} e^{4\phi_4} \left\{|\Theta_a| + \frac{\mathcal{D}_a^2}{2|\Theta_a|} + \frac{e^{-2\phi}||\ell_i||^2|\mathcal{Q}_a|^2}{2|\Theta_a|} + \cdots\right\} \tag{5.18}$$

This expansion supports the interpretation of the supersymmetry constraints $\text{Im}\left(\mathcal{F} \wedge \Omega\right) = 0$ and $(\mathcal{F} + iJ)^2 = 0$ as D-flatness and F-flatness conditions. On the other hand, imposing the supersymmetry conditions leaves only the first term in (5.18). In fact, in a supersymmetric configuration this term will cancel against the contribution of the orientifold planes given by

$$V_{\text{ori}} = -16e^{4\phi_4} e^{-\phi} ||\ell_{\text{ori}}|| \, . \tag{5.19}$$

In the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold that we are considering

$$||\ell_{\mathrm{ori}}|| = R_1 R_2 R_3 (1 + \tau_2\tau_3 + \tau_3\tau_1 + \tau_1\tau_2) \ . \tag{5.20}$$

and so it is easy to check that the RR tadpole cancellation conditions (3.33) imply the vanishing of the full potential of a supersymmetric brane setup. This is the usual statement that, by supersymmetry, NSNS tadpoles will also cancel in this compactification.

In the scalar potential (5.18) we clearly identify a D-term appearing in the standard supergravity form $V_D = \frac{1}{2}D_a D^a$, where $D_a = e^{2\phi_4}\mathcal{D}_a$ up to normalization, and the index a is raised with the gauge metric $(\mathrm{Re}\, f_{8a})^{-1}$. In the case of D6-branes, the analogous D-term corresponds to Fayet-Iliopoulos terms associated to the D-branes $\mathrm{U}(1)$ factors [38, 37] (see also [28, 39]). It is clear that the similar result should hold here, since we could obtain (5.16) by replacing the magnetized D8-brane by a non-factorizable D6-brane whose wrapping numbers are given by (3.19).

On the other hand, the scalar potential also includes an F-term contribution, namely

$$V_F = \frac{1}{16\pi^3\alpha'^2} \frac{e^{2\phi}||\ell_i||^2|\mathcal{Q}_a|^2}{\mathcal{V}^2 \mathrm{Re}\, f_{8a}} \ . \tag{5.21}$$

In the following we will argue that this F-term is due to a superpotential

$$W = \frac{\sqrt{2}}{\pi^2(\alpha')^{3/2}} X \mathcal{Q}_a \ , \tag{5.22}$$

where $\mathcal{Q}_a$ depends on closed Kähler moduli as shown in (5.16), whereas X is an open string modulus of the coisotropic D8$_a$-brane. The task is to show that V_F is of the standard form

$$e^K K^{\bar{X}X} \left|\frac{\partial W}{\partial X}\right|^2 \ , \tag{5.23}$$

where K is the Kähler potential of the closed and open moduli and, as usual, $K^{\bar{A}B}$ is the inverse of $K_{\bar{A}B} = \partial_{\bar{A}}\partial_B K$.

First, the Kähler potential has the general structure

$$K = \hat{K} + \tilde{K}_{X\bar{X}} X\bar{X} + \cdots , \tag{5.24}$$

with $\hat{K}$ and $\tilde{K}_{X\bar{X}}$ depending only on the closed moduli. For $\hat{K}$ we have the well-known tree-level result

$$\hat{K} = -\log(S + \bar{S}) - \sum_{i=1}^{3} \log(U_i + \bar{U}_i) - \sum_{i=1}^{3} \log(T_i + \bar{T}_i) \tag{5.25}$$

and so, to first order in X,

$$e^K = \frac{e^{4\phi}}{128\mathcal{V}^3} + \cdots \ . \tag{5.26}$$

Second, we need to determine $\tilde{K}_{X\bar{X}}$, and our strategy will be to compute the kinetic energy of the open string modulus, i.e. $\tilde{K}_{X\bar{X}}\partial_\mu X \partial^\mu \bar{X}$, from the Dirac-Born-Infeld action.

Let us first identify the open string modulus X. To simplify the discussion we will consider a fractional D8$_a$-brane wrapping the 5-cycle $(1,0)_1 \times ((\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3)/\mathbb{Z}_2$ (see section 3.2). Then, the obvious moduli of our D8-brane in the D8$_a$D8$_a$ sector are the position y^1 of $(1,0)_1$ in $(\mathbf{T}^2)_1$, and the Wilson line A_1 along the unique 1-cycle $(1,0)_1$. These two real moduli can be arranged into a complex field as

$$X = R_1^2 \tau_1 \, y^1 + i A_1 \tag{5.27}$$

where the factor of $\mathrm{Re}\, T_1$ in $\mathrm{Re}\, X$ is necessary in order to obtain a canonical kinetic energy. In fact, upon reducing the Dirac-Born-Infeld action the terms quadratic in space-time derivatives of y^1 and A_1, in Einstein frame, turn out to be

$$\frac{1}{4\pi\alpha'} \frac{e^{2\phi}}{\mathcal{V}} \sqrt{\tilde{g}} \, \mathrm{Re}\, f_{8a} \left(R_1^2 \tau_1^2 \partial_\mu y^1 \partial^\mu y^1 + R_1^{-2} \partial_\mu A_1 \partial^\mu A_1 \right) \tag{5.28}$$

where the factor of $\mathrm{Re}\, f_{8a}$ basically comes from the integral over the 5-cycle that can be performed using (5.7) plus the supersymmetry constraints.

Then, from the kinetic terms we obtain

$$\tilde{K}_{X\bar{X}} = \frac{1}{4\pi\alpha'} \frac{e^{2\phi}}{\mathcal{V}||\ell_i||^2} \mathrm{Re}\, f_{8a} \ , \tag{5.29}$$

since, in the case at hand, $||\ell_i||^2 = R_1^2$. This is enough to check that the scalar potential V_F has the expected form (5.23). Indeed, collecting previous results we find

$$e^K K^{\bar{X}X} = \frac{\pi\alpha'}{32} \frac{e^{2\phi}||\ell_i||^2}{\mathcal{V}^2 \mathrm{Re}\, f_{8a}} + \cdots \ , \tag{5.30}$$

to first order in X. Finally, the proposed superpotential obviously verifies $\frac{\partial W}{\partial X} \sim \mathcal{Q}_a$.

To summarize, we find that for a coisotropic D8-brane (3.18) the worldvolume flux F induces a superpotential linear in its open string modulus X_i which, as in the case of D6-branes, is a combination of transverse position and Wilson line in $(\mathbf{T}^2)_i$. Such superpotential also involves the Kähler moduli of the compactification, and has the general form

$$W = X_i(T_j T_k - n) \tag{5.31}$$

where n comes from integrating F^2 over $(\mathbf{T}^2)_j \times (\mathbf{T}^2)_k$. Such type of superpotentials have also been obtained in the case of magnetized D7-branes [40, 41].

In general, the existence of a open-closed string superpotential is quite an interesting feature of a D-brane model and, in principle, it may provide new sources of moduli stabilization. Naively, one would say that in the presence of a superpotential of the form (5.31) the Kähler moduli should be fixed as $T_j T_k = n$, which is precisely the F-flatness condition for our D8-brane. One should be however careful, because there could be extra superpotential terms involving X_i and then this naive picture could be complicated. For instance, we have analyzed the open string modulus X_i, which exists for any fractional D8$_a$-brane on $(n_a^i, m_a^i)_i \times ((\mathbf{T}^2)_j \times (\mathbf{T}^2)_k)/\mathbb{Z}_2$. However, there could be extra open string moduli, say X_j and X_k, coupling as

$$W_X = X_i X_j X_k \tag{5.32}$$

as is the case for D6-branes [26, 4]. Then the F-flatness condition for X_i would read

$$\frac{\partial(W + W_X)}{\partial X_i} = (T_j T_k + X_j X_k - n) = 0 \tag{5.33}$$

and hence the deviation of the Kähler moduli from the BPS condition could be compensated by giving a vev to open string moduli. Notice that this phenomenon is quite analogous to the case of D-terms for D6-branes, where the scalar potential does not fix the complex structure moduli of the compactification, but only a combination of closed and open string moduli. In this sense, the fact that $X_j X_k$ take a vev could be interpreted as some kind of D8-brane recombination.

Notice also that, as a byproduct of our analysis, we have determined the Kähler metric of the open string field X. This metric has a very simple expression as function of the closed moduli. For instance, for a D8$_a$-brane wrapping $(1,0)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3$ we find

$$\tilde{K}_{X\bar{X}} = \frac{1}{4\pi\alpha' \operatorname{Re} T_1} \left(\frac{n_a^{xx}}{\operatorname{Re} S} - \frac{n_a^{yy}}{\operatorname{Re} U_1} \right) . \tag{5.34}$$

If the 5-cycle is instead $(0,1)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3$ the metric is given by

$$\tilde{K}_{X\bar{X}} = -\frac{1}{4\pi\alpha' \operatorname{Re} T_1} \left(\frac{n_a^{xy}}{\operatorname{Re} U_2} + \frac{n_a^{yx}}{\operatorname{Re} U_3} \right) . \tag{5.35}$$

In general, we just need to apply eq. (5.29). For magnetized D7-branes the open moduli metric has been discussed in [42–44, 40, 41].

5.3 Massive $\mathrm{U}(1)$'s

The axionic coupling (5.10) not only enters in the gauge kinetic function of the effective theory, but it is also an important ingredient in the cancellation of $\mathrm{U}(1)$ anomalies through a generalized Green-Schwarz mechanism [45]. One can see that the imaginary parts of the dilaton ($S \equiv U_0$) and the complex structure moduli couple to the $D = 4$ gauge field strengths as
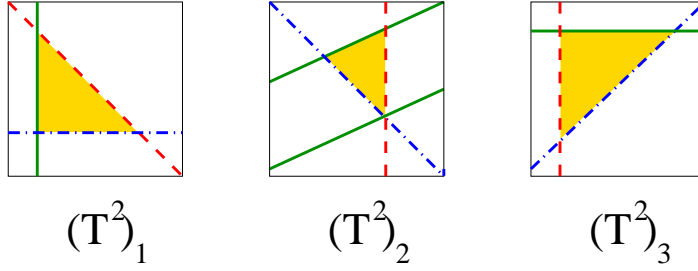
$$\sum_{L=0}^{3} d_L^a \operatorname{Im} U_L \operatorname{Tr} (F_a \wedge F_a) . \tag{5.36}$$

For instance, if F_a arises from the coisotropic D8$_a$, with data given in (3.18) and $i = 1$, we find the coefficients

$$d_L^a = (-n_a^1 n_a^{yy}, \; -n_a^1 n_a^{xx}, \; m_a^1 n_a^{yx}, \; m_a^1 n_a^{xy}) . \tag{5.37}$$

In order to cancel $\mathrm{U}(1)$ anomalies we also need $B \wedge \operatorname{Tr} F_a$ couplings, which can also be obtained from the Chern-Simons action. Indeed, from (5.6) we obtain couplings of the form

$$\operatorname{Tr} F_a \wedge \int_{\Pi_5^a} \mathcal{F} \wedge C_5 = \sum_{L=0}^{3} c_L^a B_L \wedge \operatorname{Tr} F_a . \tag{5.38}$$

Figure 7: Yukawa couplings in intersecting D6-branes involve a sum over triangle shaped instantons.

where the B_L are 2-forms dual to the 0-forms $\mathrm{Im}\,U_L$. In the example above ($i = 1$) the coefficients c_L^a are given by

$$c_L^a = (-N_a m_a^1 n_a^{xx},\ -N_a m_a^1 n_a^{yy},\ N_a n_a^1 n_a^{xy},\ N_a n_a^1 n_a^{yx})\ . \tag{5.39}$$

Obviously, these couplings exist only for Abelian $\mathrm{U}(1)$ factors.

The couplings (5.36) and (5.38) give a contribution

$$\sum_L c_L^i d_L^a\ , \tag{5.40}$$

to mixed and cubic $\mathrm{U}(1)_a$ anomalies. Once the RR tadpole conditions (3.33) have been imposed, this piece will cancel the remaining triangle anomalies due to the charged massless fermions in the spectrum. In fact, the $\mathrm{U}(1)$-$\mathrm{SU}(N)^2$ anomaly cancellation works exactly like in the case of D6-branes, so we refer the reader to the original references [2, 46] for a detailed computation.

As usual, when the terms $B \wedge \mathrm{Tr}\,F_a$ do not vanish, there will be combinations of $\mathrm{U}(1)$ factors that acquire a mass. A generic $\mathrm{U}(1)$ generator $Q = \sum_i \xi_i Q_i$, where Q_i arises from the ith stack, will remain massless provided that

$$\sum_i c_L^i \xi_i = 0 \quad ; \quad \forall L \tag{5.41}$$

which guarantees that the corresponding $\mathrm{U}(1)$ does not couple to any RR 2-forms B_L.

5.4 Yukawa couplings

Other effective field theory quantities of obvious interest are the Yukawa couplings among chiral fields. In the case of intersecting D6-branes, the flavor-dependent contribution to the Yukawa couplings is given by a sum over worldsheet instantons [2] (see figure 7). The couplings among three chiral fields labeled α, β, γ, have the qualitative form

$$Y_{\alpha\beta\gamma} \ \propto\ \prod_i \left(\sum_{k_i \in \mathbb{Z}} e^{-\frac{A_{\alpha\beta\gamma}(k_i)}{2\pi\alpha'}} \right) \tag{5.42}$$

– 30 –
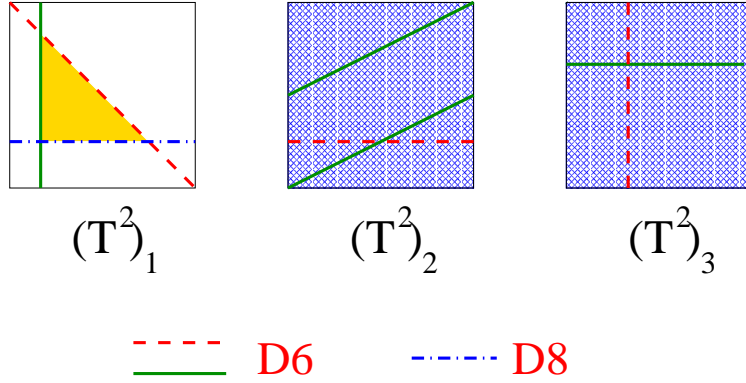
Figure 8: Yukawa couplings involving two D6-branes and one D8 contain contributions both from instantons (first torus in the figure) and integrals over overlapping wave functions.

where $i = 1, 2, 3$, labels the three tori and $A_{\alpha\beta\gamma}(k_i)$ is the area of the triangular instantons on each of them. This sum was shown in [7] to be equal to a product of Jacobi theta-functions in the toroidal/orbifold case. The type I mirror of these orientifolds consists of D5-branes and magnetized D9-branes, and the Yukawa couplings come from a pure field-theoretical calculation. They are given by the overlap integral over the 6 extra dimensions of the three wave functions of the zero modes α, β, γ, i.e.,

$$Y_{\alpha\beta\gamma} \propto \int_{(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3} d^6 y \ \Psi_\alpha(y)\Psi_\beta(y)\Psi_\gamma(y) \tag{5.43}$$

where some of the wavefunctions are delta functions when D5-branes are involved in the Yukawa coupling. As checked in [47], this computation yields the expected T-dual of the computation in terms of intersecting D6-branes.

In the presence of coisotropic D8-branes we find three new different classes of brane configurations giving rise to Yukawa couplings among chiral fields. In these cases, the origin of Yukawa couplings turns out to be a combination of the above two mechanisms. The three new classes are as follows:

(a) *Yukawa couplings involving two* D6*'s and one* D8. This is depicted in figure 8 for the case of a D8-brane with the 1-cycle in the first torus. The Yukawa coupling will have two factors, one contribution coming from a sum over triangle instantons in the first torus (see the figure) and the other from the overlap integral of wave functions over the second and third torus. One thus has a structure for the couplings of the form

$$Y_{\alpha\beta\gamma} \propto \left(\sum_{k_1} e^{-\frac{A_{\alpha\beta\gamma}(k_1)}{2\pi\alpha'}}\right) \times \int_{(\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3} d^4 y \ \Psi_\alpha(y)\Psi_\beta(y)\Psi_\gamma(y) \tag{5.44}$$

where the wavefunctions have only support in the D-brane intersections. In particular, the wavefunction arising from the D6D6 sector will be a delta function on $(\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3$.
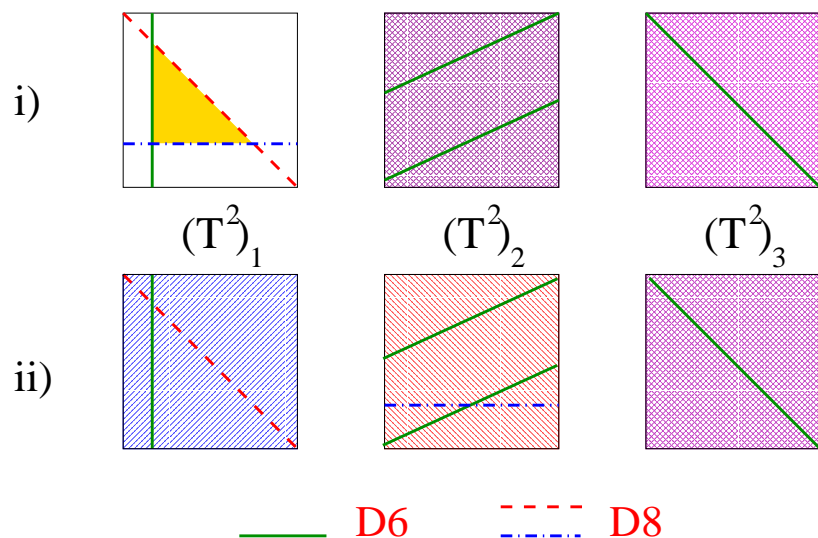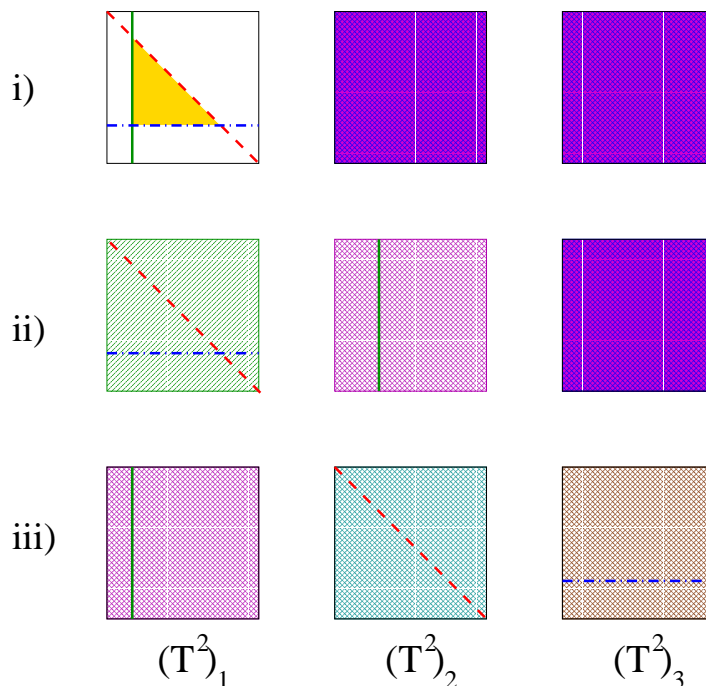
Figure 9: The two general classes of Yukawa couplings in configurations with two D8's and one D6. In the first one there are contributions from a sum over instantons and overlap integrals. In the second case there is no sum over instantons.

- (b) *Yukawa couplings involving two* D8*'s and one* D6. This is depicted in figure 9. As shown in the figure, there are two classes of configurations of this type. In the first the 1-cycle of both D8-branes are in the same torus, i.e. $(\mathbf{T}^2)_1$. In this case the structure of the Yukawa couplings is similar to the previous case and the Yukawa coupling has the general form in eq. (5.44). In the second case in the figure there is no sum over instantons and the structure is rather as in eq. (5.43).

- (c) *Yukawa couplings involving three* D8*'s.* In this case there are three possible configurations of the D8's, as shown in figure 10. In the first configuration again we have a sum over instantons in one of the tori ($(\mathbf{T}^2)_1$ in the figure) and the Yukawa coupling will be given by an expression as in eq. (5.44). In the other two cases we will have an expression as in eq. (5.43).

It would be interesting to see which kind of textures yield these new classes of Yukawa couplings. However, a detailed computation of these Yukawa couplings goes beyond the scope of this paper, and is left for future work.

6. Coisotropic model building

The coisotropic D8-branes discussed in the previous sections turn out to be interesting from the model building point of view. One of the reasons is that coisotropic D8-branes have induced D6-brane charges corresponding to non-factorizable 3-cycles. This allows for new ways to cancel RR tadpoles in constructions giving rise to a MSSM-like spectrum. In addition the presence of an appropriate set of coisotropic D8-branes give rise to super-potential couplings involving the Kähler moduli and may be useful in order to fix them.

Figure 10: The three general classes of Yukawa couplings involving three D8's.

We postpone for future work a systematic analysis of the model-building possibilities of this new tool, but we will present here two examples of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold models with both D8- and D6-branes and a chiral spectrum close to that of a 3-generation MSSM-like model. In the first example the MSSM fields will reside on a set of intersecting D6-branes and the addition of a stack of coisotropic D8-branes will ensure RR tadpole cancellations. In the second example the MSSM fields will reside at the intersection of both D8- and D6-branes and extra coisotropic D8-branes will be added both to cancel tadpoles and provide superpotential couplings for the untwisted Kähler moduli.

6.1 An MSSM-like model

The first model will be based on one of the 'triangle quiver' toroidal models discussed in section 4.4 of ref. [23], concretely, that with $\beta^2 = 1$ (see that reference for details). In this model the third torus is not a square but a 'tilted' torus. The extension of this set of D6-branes to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold case is simply done by doubling the number of D6-branes as in [4]. We thus have D6-branes with wrapping numbers as in table 4.

One can easily check that the set of D6-branes in this model are not enough to cancel RR-tadpoles. In particular, their contribution to each of the 4 tadpoles is respectively $(76, -4, -2, -4)$ whereas that of the orientifold planes is $(-16, 8, 8, 16)$.[10] In order to cancel tadpoles we need branes contributing charges as $(-60, -4, -6, -12)$. A very economical possibility is to add a stack of 4 coisotropic D8-branes M (and their corresponding images

---

[10]Note that the second and third orientifold charges are halved due to the tilting of the third torus.

N_α Dp_α	D8$_\alpha$: $(n^i_\alpha, m^i_\alpha)_i \times (n^{xx}_\alpha, n^{xy}_\alpha, n^{yx}_\alpha, n^{yy}_\alpha)_{jk}$
	D6$_\alpha$: $(n^1_\alpha, m^1_\alpha)_1 (n^2_\alpha, m^2_\alpha)_2 (n^3_\alpha, m^3_\alpha)_3$
$N_a = 6$ D6$_a$	$(1,0)_1(3,1)_2(3,-1/2)_3$
$N_b = 4$ D6$_b$	$(1,1)_1(1,0)_2(1,-1/2)_3$
$N_c = 2$ D6$_c$	$(0,1)_1(0,-1)_2(2,0)_3$
$N_d = 2$ D6$_d$	$(1,0)_1(3,1)_2(3,-1/2)_3$
$N_M = 4$ D8$_X$	$(-1,-3,-2,-5)_{12} \times (-3,1/2)_3$

Table 4: Set of D6 and coisotropic D8-branes giving rise to the MSSM-like model in the text.

under the orientifold operation) as shown in the table. One can easily check that indeed this simple addition cancels all tadpoles. All D-branes in the model preserve the same $\mathcal{N}=1$ SUSY for appropriate choices of the complex structure moduli. In particular the D-term conditions are

$$2\tau_1 = 2\tau_2 = \tau_3 \quad ; \quad \tau_1\tau_2\tau_3 = 18\tau_1 + 12\tau_2 + 5\tau_3 \tag{6.1}$$

which are obeyed for $\tau_1 = \tau_2 = \tau_3/2 = \sqrt{20}$. A superpotential of the form

$$W_{D8} = X_3(1 - T_1T_2) \tag{6.2}$$

is created due to the presence of the D8-brane. In absence of the superpotential (5.32) or for fixed extra D8-brane moduli, this would constrain the $T_{1,2}$ Kähler moduli to satisfy $T_1T_2 = 1$.

The gauge group is initially $U(3)_c \times U(2)_L \times U(1)_{B-L} \times U(1)_R \times U(2)_M$. However, three out of the 5 U(1)'s are anomalous and only the hypercharge and an additional U(1) (the one characteristic of left-right symmetric models) survives the $B \wedge F$ couplings described in the last section, remaining as massless U(1)'s. The chiral spectrum is shown in table 5 and consists of the content of the MSSM plus some additional SM doublets and singlets. Further antisymmetric and symmetric $U(2)_M$ chiral fields uncharged under the SM exist which we do not display.

This model, which is of phenomenological interest by itself, exemplifies how coisotropic D8-branes may be a useful tool for model-building purposes. In this example the role of the D8 was auxiliary, in the sense that it helped in cancelling RR tadpoles and restricting Kähler moduli but the MSSM fields reside on D6-branes. Models analogous to this in which MSSM fields live on D8-branes can also be constructed. This is illustrated in the next example in which also additional D8-branes are appended in order to provide superpotential couplings for the untwisted Kähler moduli.

6.2 A left-right symmetric MSSM-like model

In our second example the brane structure is reminiscent of the MSSM-like D6-brane configuration introduced in [6, 7] and first included in a tadpole free global $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold compactification in [5]. Consider a D8-D6 brane configuration as listed in table 6.

Intersection	Matter fields		Q_a	Q_b	Q_c	Q_d	Q_M	Q_Y
ab	Q_L	$(3,2)$	1	-1	0	0	0	1/6
ab'	q_L	$2(3,2)$	1	1	0	0	0	1/6
ac	U_R	$3(\bar{3},1)$	-1	0	1	0	0	-2/3
ac'	D_R	$3(\bar{3},1)$	-1	0	-1	0	0	1/3
bd	L	$(1,2)$	0	-1	0	1	0	-1/2
bd'	l	$2(1,2)$	0	1	0	1	0	-1/2
cd	N_R	$3(1,1)$	0	0	1	-1	0	0
cd'	E_R	$3(1,1)$	0	0	-1	-1	0	1
bc	H	$(1,2)$	0	-1	1	0	0	-1/2
bc'	$\bar{H}$	$(1,2)$	0	-1	-1	0	0	1/2
bM	F	$3(1,2;2_M)$	0	-1	0	0	1	0
bM^*	$\bar{F}$	$2(1,2;2_M)$	0	1	0	0	1	0
cM	G	$5(1,1;2_M)$	0	0	-1	0	1	1/2
cM^*	$\bar{G}$	$5(1,1;2_M)$	0	0	1	0	1	-1/2

Table 5: Chiral spectrum of the MSSM-like model discussed in the text. The hypercharge generator is defined as $Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d$.

$N_\alpha \ \mathrm{D}p_\alpha$	$\mathrm{D8}_\alpha: \ (n_\alpha^i, m_\alpha^i)_i \times (n_\alpha^{xx}, n_\alpha^{xy}, n_\alpha^{yx}, n_\alpha^{yy})_{jk}$ $\mathrm{D6}_\alpha: \ (n_\alpha^1, m_\alpha^1)_1 (n_\alpha^2, m_\alpha^2)_2 (n_\alpha^3, m_\alpha^3)_3$
$N_a = 6 + 2 \ \mathrm{D8}_a$	$(1,0)_1 \times (1,3,-3,-10)_{23}$
$N_b = 2 \ \mathrm{D6}_b$	$(0,1)_1(1,0)_2(0,-1)_3$
$N_c = 2 \ \mathrm{D6}_c$	$(0,1)_1(0,-1)_2(1,0)_3$
$N_M = 4 \ \mathrm{D6}_M$	$(-2,1)_1(-3,1)_2(-3,1)_3$
$N_X = 2 \ \mathrm{D8}_X$	$(1,0)_2 \times (1,0,0,-2)_{31}$
$N_Y = 2 \ \mathrm{D8}_Y$	$(1,0)_3 \times (1,0,0,-2)_{12}$

Table 6: Set of D6 and coisotropic D8-branes giving rise to the left-right symmetric MSSM-like model in the text.

The stacks of branes a, b, c, contain the SM gauge group and particles whereas the stacks M, X, and Y are auxiliary branes whose mission is helping in cancelling tadpoles. In addition the D8's X and Y also contribute to the fixing of the three untwisted Kähler moduli T_i. Note that the coisotropic D8-branes involved in the model have induced D6-

Intersection	Matter fields		Q_a	Q_d	Q_M
ab	Q_L	$3(3, 2, 1)$	1	0	0
ac	Q_R	$3(\bar{3}, 1, 2)$	-1	0	0
bd	L	$3(1, 2, 1)$	0	-1	0
cd	R	$3(1, 1, 2)$	0	1	0
bc	H	$(1, 2, 2)$	0	0	0
bM	F	$6(1, 2, 1; 2_M)$	0	0	-1
cM	G	$6(1, 1, 2; 2_M)$	0	0	-1

Table 7: Chiral spectrum of the MSSM-like left-right symmetric model discussed in the text.

charge given by a sum of two factorized cycles:

$$\text{D8}_a : (1,0)_1 \times (1, 3, -3, -10)_{23} = (1,0)_1 \times [(3,1)(3,-1) + (1,0)(1,0)]$$
$$\text{D8}_X : (1,0)_2 \times (1, 0, 0, -2)_{31} = (1,0)_2 \times [(0,1)(0,-1) + (2,0)(1,0)]$$
$$\text{D8}_Y : (1,0)_3 \times (1, 0, 0, -2)_{12} = (1,0)_3 \times [(0,1)(0,-1) + (2,0)(1,0)]$$

One can easily check that all RR-tadpoles cancel and that, for appropriate choices of closed string moduli, all these branes preserve the same $\mathcal{N}=1$ SUSY as the orientifold background. The D8-branes a and d give rise to a group $\text{SU}(3)_c \times \text{U}(1)_{B-L} \times \text{U}(1)_{B+L}$, where the latter U(1) is anomalous and becomes massive as usual through the $D = 4$ Green-Schwarz mechanism. The D6-branes b and c sit on top of the horizontal orientifold plane and carry symplectic groups, in the case at hand one has $\text{USp}(2)_L \times \text{USp}(2)_R$ corresponding to a left-right symmetric gauge group $\text{SU}(2)_L \times \text{SU}(2)_R$. Finally, the D8-brane M gives rise to an extra non-Abelian factor $\text{U}(2)_M$.

It is easy to compute the chiral spectrum in this model and find that the chiral spectrum with respect to the gauge group $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$ has quantum numbers as shown in table 7, i.e. three generations of quarks and leptons. They correspond to open strings in between the branes a, d and b, c. In addition there are exotic leptons/Higgsses $6(2_M, 2_L) + 6(2_M, 2_R)$ corresponding to open strings in between the brane M and b, c. However, such exotic matter can be higgsed away by giving a vev to the symmetric or to the antisymmetric fields charged under $\text{U}(2)_M$, and which arise from open strings stretched between M and its orientifold image M'. From the field theory point of view, these vevs trigger the gauge symmetry breaking $\text{U}(2)_M \to \text{SO}(2)_M$ and $\text{U}(2)_M \to \text{SU}(2)_M$, respectively, after which $6(2_M, 2_L) + 6(2_M, 2_R)$ can acquire a mass. From a geometric point of view, such process corresponds to a D-brane recombination $M + M' \to \tilde{M}$, and one can see that the intersection product of $\tilde{M}$ with the branes b and c vanishes. Finally, such D-brane recombination occurs spontaneously in a large region of the complex structure moduli space, as can be deduced from the FI-terms of this model.

Notice that a, b, c, d brane structure is analogous to the MSSM-like model constructed in [6, 7, 5]. Similar to the latter, the MSSM Higgs sector arises from the open strings stretched between the D-branes b and c, which overlap in the first complex dimension and hence give rise to a non-chiral sector of the spectrum. In addition to the above chiral fields, there are further SM singlets which we do not display.

The D-term conditions give in the present example

$$\tau_2 = \tau_3 \quad ; \quad \tau_1\tau_2\tau_3 = 9\tau_1 + 6\tau_2 + 6\tau_3. \tag{6.3}$$

Recall that one cannot conclude that the complex structure moduli are fixed to obey these constraints, since a deviation from these equations give rise to a FI-terms whose contribution to the vacuum energy will be cancelled by vevs of chiral matter at the intersections. In this case gauge symmetry breaking will take place corresponding to brane recombination, but only some linear combinations of complex structure moduli and matter scalars are in fact fixed. On the other hand the presence of the a, X and Y D8-branes give rise to a superpotential of the form

$$W_{D8} = X_1^a(1 - T_2T_3) + X_2^X(2 - T_1T_3) + X_3^Y(2 - T_1T_2) \tag{6.4}$$

where $X_i^{a,X,Y}$ are the geometric + Wilson line open string moduli. Note that for fixed open string moduli this would fix the three untwisted Kähler moduli to the values

$$\mathrm{Re}\, T_1 = 2 \quad ; \quad \mathrm{Re}\, T_2 = \mathrm{Re}\, T_3 = 1 \quad ; \quad \mathrm{Im}\, T_i = 0 \ . \tag{6.5}$$

One can play around with the magnetic fluxes to get other models with larger Kähler moduli but recall that, unlike the case of moduli fixing through closed string fluxes, the present models correspond to CFT's and hence we do not need to work in the large volume limit approximation.

7. Coisotropic branes and mirror symmetry

As emphasized at the introduction, coisotropic branes were first introduced in order to correctly formulate Kontsevich's Homological Mirror Symmetry conjecture. Although we have not made use of this fact anywhere along our discussion, a natural question is how these new type IIA vacua look upon applying mirror symmetry. In particular, if coisotropic D8-branes in $\mathbf{CY}_3$'s have not been considered before and they are mapped to type IIB Dp-branes by the mirror map, one may wonder if we have been missing any kind of D-brane in type IIB constructions. This question is particularly meaningful in the case of simple $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds, where the mirror map is well understood and chiral D-brane vacua have been constructed in both type IIA and type IIB sides.

In the following, we will answer this question for the type IIA $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold considered in this paper, originally introduced in terms of its type I mirror [48]. We will see that the type I duals of a coisotropic D8-branes can either be a tilted D5-brane or a magnetized D9-brane, the magnetic flux involved in the latter being rather exotic. Finally, we will make use of the web of T-dualities to show that coisotropic D8-branes are also related to a new class of special Lagrangian D6-branes in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, which have so far not been considered in the construction of $\mathcal{N} = 1$ chiral type IIA vacua.
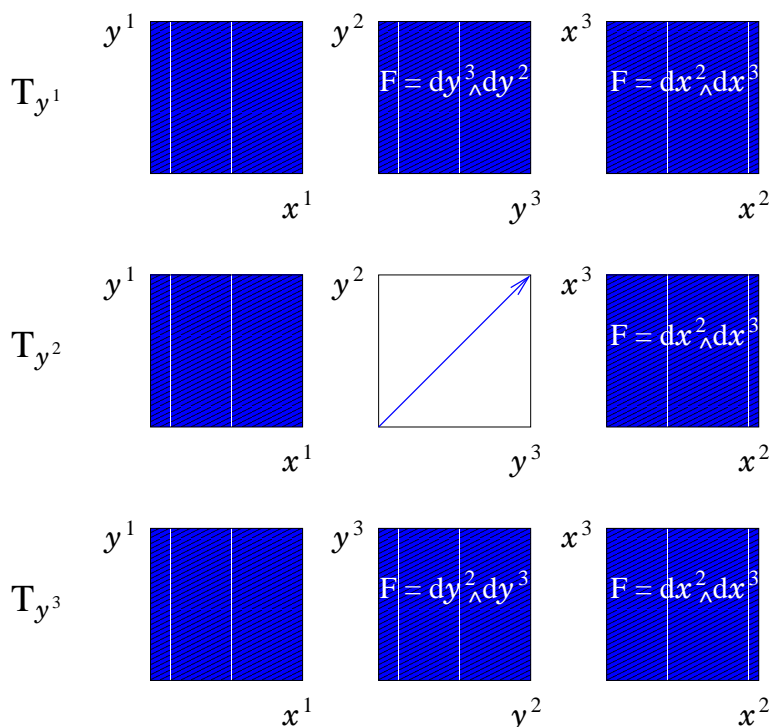
Figure 11: Mirror symmetry for the coisotropic D8-brane (7.1).

7.1 Mirror symmetry to type I

We will first illustrate how mirror symmetry works by looking at a rather simple example, so let us consider type IIA compactified on $\mathbf{T}^6$ and the BPS, coisotropic D8-brane given by

$$
\begin{aligned}
\Pi_5 &= (1,0)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3 \\
F/2\pi &= dx^2 \wedge dx^3 - dy^2 \wedge dy^3
\end{aligned}
\tag{7.1}
$$

also considered at the beginning of section 3. If we orientifold this theory by $\Omega\mathcal{R}(-1)^{F_L}$, with $\mathcal{R}$ given by (3.22), we will introduce an O6-plane on the directions $\{x^1, x^2, x^3\}$. Hence, in order to map this setup to type I theory compactified on $\mathbf{T}^6$, we need to perform three T-dualities on the directions $\{y^1, y^2, y^3\}$.

How the D8-brane transforms under these three T-dualities is illustrated in figure 11. A T-duality along y^1 is trivial and only maps the D8-brane to a D9-brane with the same worldvolume flux F. In order to perform the second T-duality we conveniently relabel the coordinates as in the figure, in order not to have components of F which mix different $\mathbf{T}^2$ factors. The T-duality along y^2 is then simply deduced from the usual map between torons and branes at angles [32], obtaining a D8-brane tilted in the $\{y^2, y^3\}$ plane. Finally, applying the same kind of map we recover a D9-brane with worldvolume flux

$$
F/2\pi = dx^2 \wedge dx^3 + dy^2 \wedge dy^3.
\tag{7.2}
$$

One of the advantages of this type I picture is that it is easy to work out the orientifold image of such D-brane. Type I theory can be simply seen as type IIB modded out by the world-sheet parity Ω, which acts in the D9-brane worldvolume flux as $\Omega : F \to -F$. Taking the image of (7.2) and undoing the previous three T-dualities we arrive to the coisotropic D8-brane

$$
\begin{aligned}
\Pi_5 &= (-1,0)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3 \\
F/2\pi &= -dx^2 \wedge dx^3 + dy^2 \wedge dy^3
\end{aligned}
\tag{7.3}
$$

in agreement with (3.23). In addition, one can understand the BPS conditions (3.7) from this mirror picture. Indeed, in the case of type IIB compactified in a Calabi-Yau and in the presence of O9-planes and/or O5-planes, the supersymmetry conditions for a D9-brane read [10]

$$
\begin{array}{lc}
\text{D} - \text{flatness} & \frac{1}{3!}\mathcal{F}^3 = \frac{1}{2}J^2 \wedge \mathcal{F} \\
\text{F} - \text{flatness} & \mathcal{F}^{(0,2)} = 0
\end{array}
\tag{7.4}
$$

While the D-flatness condition is trivial for (7.2), the F-flatness condition requires F to be a (1,1)-form on the D9-brane worldvolume. It is easy to see that, if we parameterize our complex structure moduli as $dz^j = dx^j + i\mathfrak{t}_j dy^j$ (where $\mathfrak{t}_j$ is now a complex number), this implies that $\mathfrak{t}_2 \cdot \mathfrak{t}_3 = 1$, which is the mirror condition to (3.7).

Type I compactifications on magnetized tori have been considered long ago [49]. More recently, magnetic fluxes of the form (7.2) have appeared in the context of type I theory compactified on $\mathbf{T}^6$ [50, 51]. Now, if we want to describe the mirror of our type IIA vacua on $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ we need to consider type I theory compactified on the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with opposite choice of discrete torsion, and embed our magnetized D9-brane in such background. While it may seem that this still is a close relative of the constructions in [50, 51], there is actually an important difference. Namely, the 2-form flux (7.2) is cohomologically non-trivial in the case of $\mathbf{T}^6$, but it becomes trivial for $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$. In particular, the orbifold group generator (3.24) acts as $\theta : F \to -F$ when applied to (7.2), so F is not even well-defined as a 2-form in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$. Of course, this is not a problem for our orbifold construction, because F needs to be well-defined in terms of the D9-branes, and not in terms of the closed string background.[11] On the other hand, F^2 can be seen as a well-defined, non-trivial representative of a $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ cohomology class.

One can get a better intuition of what these facts mean by looking at the induced D-brane charges of our magnetized D9-brane and, in particular, by going back to the original type IIA mirror construction. For instance, the fact that F is trivial in cohomology will imply that the induced charge of D7-brane will vanish, and this statement is mirror to the fact that our coisotropic D8-brane is wrapping a 5-cycle Π_5 trivial in homology. More

---

[11]More precisely, for each D9-brane with worldvolume flux F we will need to add a θ-image D9-brane with worldvolume flux $-F$, so that we can quotient our theory by $\mathbb{Z}_2 \times \mathbb{Z}_2$.

precisely,

$$
\begin{array}{ccc}
\text{type IIB} & & \text{type IIA} \\[4pt]
[F] \quad \text{trivial} & \longleftrightarrow & \left\{
\begin{array}{ll}
[\Pi_5] & \text{trivial} \\
[\Pi_1^{F^2}] & \text{trivial}
\end{array}
\right. \\[16pt]
\left.
\begin{array}{ll}
[\Pi_6] & \text{non} - \text{trivial} \\
[F^2] & \text{non} - \text{trivial}
\end{array}
\right\} & \longleftrightarrow & [\Pi_3^F] \quad \text{Non} - \text{trivial}
\end{array}
\tag{7.5}
$$

where Π_6 is the 6-cycle wrapped by the D9-brane plus its orientifold images. To sum up, there is a conceptual difference between embedding the worldvolume flux (7.2) in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ with respect to $\mathbf{T}^6$. Such difference is the same than constructing a coisotropic D8-brane in a homologically trivial 5-cycle of a proper $\mathbf{CY_3}$ rather than in $\mathbf{T}^6$ or in $\mathbf{T}^2 \times \mathbf{K3}$, where coisotropic branes were already known to exist.

For our purposes, however, the really interesting property of $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ with respect to $\mathbf{T}^6$ or $\mathbf{K3}$ is that we can construct $D=4$, $\mathcal{N}=1$ type I string vacua. Those vacua will contain $D=4$ chiral fermions if we introduce non-trivial worldvolume fluxes in our D9-branes, even in the case of the more exotic bundles of the form (7.2). That this is the case was proved in section 4 by means of the mirror type IIA picture, but it can also be done directly in type I by generalizing the techniques of [52] to toroidal orbifolds. We will not attempt to give such general description here, but rather illustrate how to construct new $D=4$, $\mathcal{N}=1$ chiral models in type I by describing the mirror of one of the models of the previous section.

In particular, let us consider the MSSM-like model constructed in table 6. If we first consider the Pati-Salam sector of the theory, i.e., branes a, b and c we see that only the D-brane stack a changes with respect to the local model in [7]. In the type I picture, such D8-brane becomes a stack of 10 D9-branes with worldvolume flux

$$
F_a/2\pi = \left(\frac{1}{10}(dx^2 \wedge dx^3 + dy^2 \wedge dy^3) + \frac{3}{10}(dx^2 \wedge dy^2 - dx^3 \wedge dy^3) \right) \cdot \mathbf{1}_{10}
\tag{7.6}
$$

This flux will break the initial $\mathrm{U}(10)$ gauge group down to $\mathrm{U}(1)$, via non-Abelian Wilson lines (see, e.g., [47]), so in fact we will need 4 times this D9-brane content to generate the desired $\mathrm{U}(3) \times \mathrm{U}(1)$ gauge symmetry and, when embedding our model in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, a total of 80 D9-branes will be needed. Incidentally, notice that in the case of (7.6) θ does not map F_a to $-F_a$, but only some components get 'projected out'. On the other hand, $F_a^2 = -dx^2 \wedge dy^2 \wedge dx^3 \wedge dy^3$, is well-defined and implies that a D5$_1$-brane charge is induced on the D9-brane.[12] Finally, one can check that both supersymmetry conditions in (7.4) will be satisfied once that we pick $\mathfrak{t}_2 \cdot \mathfrak{t}_3 = 1$ and $\mathrm{Area}(\mathbf{T}^2)_2 = \mathrm{Area}(\mathbf{T}^2)_3$.

The $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry of the model arises from the D-branes b and c which, in the type I picture, correspond to a D5$_2$ and a D5$_3$, respectively. Triplication of families can then be understood from the fact that

$$
\int_{D5_2} \mathrm{tr}\, F_a = -\int_{D5_3} \mathrm{tr}\, F_a = 3
\tag{7.7}
$$

---

[12]Here we are using the usual model building notation, by which D5$_i$ stands for a D5-brane wrapped on the ith complex dimension, and D7$_i$ stands for a D7-brane transverse to the same complex dimension.

and the Higgs sector of the theory will arise from open strings stretching between $D5_b$ and $D5_c$, just like in [47].

The rest of the model goes as follows. The D-brane M corresponds to 4×18 anti-D9-branes, which are nevertheless BPS because of the worldvolume flux

$$F_M/2\pi = \left(\frac{1}{2}dx^1 \wedge dy^1 + \frac{1}{3}dx^2 \wedge dy^2 + \frac{1}{3}dx^3 \wedge dy^3 \right) \cdot \mathbf{1}_{18} \tag{7.8}$$

and a certain choice of Kähler moduli. The D-branes X and Y both correspond to 2×2 D9-branes of the form

$$F_{X,Y}/2\pi = \left(dx^i \wedge dx^j + \frac{1}{2}dy^i \wedge dy^j \right) \cdot \mathbf{1}_2 \tag{7.9}$$

Finally, we need to add the images of all these D-branes under Ω.

Let us also stress that coisotropic D8-branes are not always taken to magnetized D9-branes under the mirror map. Let us consider the D8-brane

$$\begin{aligned} \Pi_5 &= (0,1)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3 \\ F/2\pi &= -dx^2 \wedge dy^3 + dx^3 \wedge dy^2 \end{aligned} \tag{7.10}$$

and again apply three T-dualities on the directions $\{y^1, y^2, y^3\}$. Following the same kind of argument than in figure 11, we obtain a D5-brane wrapping the 2-cycle

$$(1,-1)_{(x^2,y^3)} \times (1,1)_{(x^3,y^2)} \tag{7.11}$$

and with vanishing worldvolume flux F. For the particular choice of complex structure moduli $\mathfrak{t}_2 = \mathfrak{t}_3 = 1$ this becomes a holomorphic two-cycle, given by $z^2 = iz^3$, and which is the required condition for F-flatness [10]. Unlike in the case of a $D5_i$-brane, the holomorphicity of the tilted D5-brane (7.11) is non-trivial, and this eventually generates a superpotential for the complex structure moduli. In general, one can see that (7.11) is holomorphic whenever $\mathfrak{t}_2 \cdot \mathfrak{t}_3 = 1$, as we would expect from the mirror BPS conditions.

7.2 Back to type IIA

An interesting property of the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold considered in this paper is that by performing four T-dualities of the form $\{x^i, y^i, x^j, y^j\}$ we recover the same closed string background. This means that if we construct a BPS D-brane in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ and we apply these four T-dualities, we will obtain a new BPS D-brane in the same orientifold, although the values of the closed string moduli may now be different.

We would like to apply this observation to the coisotropic D8-branes constructed in this paper, in order to see if new BPS D-branes can be constructed. Let us then again consider the BPS D8-brane (7.10) and apply four T-dualities in the coordinates $\{x^1, y^1, x^2, y^2\}$. A similar reasoning than the one carried in figure 11 takes us to a D6-brane wrapping the 3-cycle[13]

$$\Pi_3 = (1,0)_{(x^1,y^1)} \times (1,1)_{(x^2,y^3)} \times (1,1)_{(x^3,y^2)} \tag{7.12}$$

---

[13]See [53] for a detailed discussion on similar T-duality computations.

which, despite being a BPS D6-brane, is not one of the usual D6-branes at angles considered in the literature. Indeed, by appropriately choosing the closed string moduli of $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, (7.12) will be related to the O6-planes by means of a SU(3) rotation. Away from this point (7.12) will no longer be a special Lagrangian, but not because it is not calibrated by $\mathrm{Re}\,\Omega$, but instead because it is no longer a Lagrangian 3-cycle. That is, just like coisotropic D8-branes, the D6-branes of the form (7.12) develop non-trivial F-terms via the failure of the BPS conditions (2.2). Given the complexified Kähler form (3.3) it is easy to see that the F-flatness condition is satisfied whenever the untwisted Kähler moduli satisfy $T_2 = T_3$. Notice that, unlike the case of coisotropic D8-branes, this is compatible with the limit where all Kähler moduli are arbitrarily large.

Notice that (7.12) by itself is not well-defined in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, but that we need to include its image under θ in order to have an invariant object under the orbifold group. The image 3-cycle is given by

$$\theta(\Pi_3) \,=\, (1,0)_{(x^1,y^1)} \times (1,-1)_{(x^2,y^3)} \times (1,-1)_{(x^3,y^2)} \tag{7.13}$$

which will be a special Lagrangian if (7.12) is. Of course, when embedded in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, (7.12) and (7.13) should be thought of 3-cycles on the covering space $\mathbf{T}^2 \times \mathbf{K3}$. In any case, one can see that the homology class of the union of both 3-cycles is given by

$$\begin{aligned}
[\Pi_3 \cup \theta(\Pi_3)] &= [(1,0)_1] \otimes [(1,0)_2(1,0)_3 + (0,1)_3(0,1)_2] \\
&= [(1,0)_1] \otimes [(1,0)_2(1,0)_3 + (0,1)_2(0,-1)_3] \tag{7.14}
\end{aligned}$$

which is exactly the D6-brane charge carried by the coisotropic D8-brane (7.1). Notice that both D-branes (7.1) and (7.12) are mutually BPS only for the particular choice of Kähler moduli $T_2 = T_3 = 1$.

To sum up, by applying four T-dualities on a coisotropic D8-brane we have found a new class of special Lagrangian D6-branes, whose 'special' condition is always satisfied but its Lagrangian condition is not. This is quite remarkable, given the fact that we usually encounter the opposite behavior for D6-branes intersecting at angles. Just like in the case of coisotropic D8-branes, this exotic D6-brane carries a D6-brane charge which is a sum of two factorizable 3-cycles (i.e., those of the form $(n^1,m^1)_1 \times (n^2,m^2)_2 \times (n^3,m^3)_3$) while nevertheless being an exact CFT boundary state. It is clear that one can construct many more D6-branes of this kind, so it would be interesting to see which new model building possibilities are opened up by the existence of these less conventional D6-branes.

8. Conclusions and outlook

Type IIA $D{=}4$ chiral vacua constructed up to now have been mostly based on intersecting D6-branes wrapping special Lagrangian 3-cycles. In this paper we have argued that in type IIA Calabi-Yau orientifolds there are other BPS objects, namely D8-branes wrapping coisotropic 5-cycles, which have so far been neglected and which seem to have interesting model-building features. These branes wrap 5-cycles in the $\mathbf{CY}_3$ which are trivial in homology, but still are stable BPS objects due to the D6-brane charge induced by magnetic fluxes

on their worldvolume. At the intersection of a D8 with another D8 or a D6 chiral fermions appear, chirality being generated by a combination of intersecting/magnetized brane mechanisms. Obviously, this new way of creating D=4 chirality makes coisotropic D8-branes an interesting tool for constructing new phenomenologically relevant string vacua.

We have analyzed in detail the case where our $\mathbf{CY}_3$ is given by a $\mathbb{Z}_2 \times \mathbb{Z}_2$ type IIA orientifold. We have worked out the chiral spectrum and effective action (gauge kinetic function, scalar potential, FI-terms, U(1) anomaly cancellation) for sets of coisotropic D8-branes and additional D6-branes in this background, and we have also studied the general form of Yukawa couplings among chiral fields. These couplings are generated by a combination of wave-function overlapping (à la Kaluza-Klein) and world-sheet instanton contributions, which might furnish new possibilities for the flavor dependence of Yukawa couplings in realistic compactifications. Thus, it would clearly be interesting to make a general exploration of patterns of Yukawa couplings in realistic models including coisotropic D8-branes. Another interesting feature of the D8-branes in this orientifold is that they carry a D6-brane charge which is not of the factorized form (3-cycle) = (1-cycle) × (1-cycle) × (1-cycle) in the underlying $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$. This opens new model-building possibilities not available to standard factorized D6-branes in this orientifold.

The presence of coisotropic D8-branes leads to superpotential couplings linear in open string moduli X_i and bilinear in untwisted Kähler moduli of the general form $X_i(T_j T_k - n)$. These couplings could be useful in order to fix Kähler moduli in specific models. In fact in the above orientifold model one could think that adding an appropriate set of coisotropic D8-branes one can actually fix all untwisted Kähler moduli. One must be careful, though, since in general there may be additional couplings of X_i to other open string moduli and the minimum of the potential could only fix a linear combination of Kähler and open string moduli, rather than just fixing the Kähler moduli. This is reminiscent of what happens with D-terms from D6-branes (and also D8-branes) which do not fix the complex structure moduli but rather a linear combination of those and charged chiral scalars. On the other hand it is clear that in the presence of other sources of moduli superpotentials (like e.g. closed string RR and/or NS fluxes) the existence of these D8-brane-induced superpotentials could be quite useful in the general moduli stabilization program. In this connection it would be interesting to study the effect of RR fluxes, which are known to lead to additional Kähler-moduli dependent superpotentials, in models with coisotropic D8-branes. In a more speculative mode, other possible use of these D8-branes could be to help in supersymmetry breaking and the up-lifting of the AdS vacua. Indeed, the Kähler moduli-dependent superpotential induced by D8-branes are reminiscent of O'Rafertaigh superpotentials. An appropriate combination of D8-branes could thus help in the generation of a (possibly metastable) SUSY-breaking vacuum. This possibility should be worth studying.

We have also found that the coisotropic D8-branes that we construct in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold have well defined type I T-duals. They correspond either to certain tilted D5-branes or D9-branes with off-diagonal fluxes. One could also T-dualize to the case of type IIB with O3/O7-planes. This could be interesting in order to make contact with the kind of models with closed string RR and NS fluxes which have been considered in order to

fix all moduli in IIB orientifolds. On the other hand we have found that four T-dualities convert certain coisotropic D8-branes into a new class of BPS D6-branes not previously considered in the model-building literature. Interestingly enough, unlike the standard factorized D6-branes considered up to now, these D6-branes give rise to non-trivial F-term conditions of the type $T_i = T_j$ in these orientifolds and have also non-factorized RR D6-charge. Consideration of this new class of D6-branes could also be interesting from the model-building point of view and is at present under study.

As an illustration of the model-building properties of coisotropic D8-branes we have presented two examples with a chiral spectrum very close to that of the MSSM, which are phenomenologically interesting in their own right. In the first model the MSSM branes are made of D6-branes and a stack of D8-branes helps in cancelling all RR tadpoles in a more efficient way that D6-branes would do. The second example provided is a D8-brane relative of the left-right symmetric D6-brane model constructed in [6, 7, 5]. A general statistical analysis of how easy it is to find consistent models of this second type and how many have an MSSM-like spectrum in the present $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold constructions was performed in [54], and more recently in [55]. It is clear that if we take into account the freedom of adding coisotropic D8-branes in these constructions new possibilities (like the two MSSM-like examples discussed in this paper) will appear. Thus, presumably the landscape of MSSM-like models within the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold will be wider than we previously thought.

Acknowledgments

A. Formal definition of coisotropic branes

For completeness, in this appendix we state the formal definition of coisotropic A-brane, as originally described by Kapustin and Orlov. Our intention is not to give a full report on this subject, but rather to provide the basic definitions that connect our results with the topological A-brane literature. For a more detailed discussion we refer the reader to the original reference [8] and related literature [56, 19, 57, 15].

Let us consider a symplectic manifold $\mathcal{M}$, so that it contains a globally well-defined, non-degenerate, closed two-form J. Given a vector field X on $\mathcal{M}$ we can map it to a 1-form ξ_X by the usual contraction of indices

$$\xi_X = \iota_X J \tag{A.1}$$

and, because of the properties of J, this map is an isomorphism. This means that we can define an inverse, J^{-1}, which maps 1-forms to vectors and, in general, covariant indices to contravariant indices.

Let us now consider a submanifold $\mathcal{N} \subset \mathcal{M}$ and define

$$\text{Ann}\, T_{\mathcal{N}} \,=\, \{\xi \in T_{\mathcal{M}}^* \mid \xi_\rho X^\rho = 0,\, \forall X \in T_{\mathcal{N}}\} \tag{A.2}$$

where $T_{\mathcal{N}}$ is the tangent bundle of $\mathcal{N}$. We then have the following definitions:

- If J takes $T_{\mathcal{N}}$ inside $\text{Ann}\, T_{\mathcal{N}} \Rightarrow \mathcal{N}$ is an isotropic submanifold

- If J^{-1} takes $\text{Ann}\, T_{\mathcal{N}}$ inside $T_{\mathcal{N}} \Rightarrow \mathcal{N}$ is a coisotropic submanifold

- If both conditions are satisfied $\Rightarrow \mathcal{N}$ is a Lagrangian submanifold

One can easily apply these conditions to the examples of coisotropic submanifolds given in the main text. If, for instance, we consider the D8-brane (3.5) then one has

$$\begin{aligned} \Pi_5 &= (1,0)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3 \\ T_{\Pi_5} &= \langle X^1, X^2, Y^2, X^3, Y^3 \rangle \\ \text{Ann}\, T_{\Pi_5} &= \langle dy^1 \rangle \end{aligned} \tag{A.3}$$

and so, because J is given by (3.3), $J(\text{Ann}\, T_{\Pi_5}) = \langle X^1 \rangle \subset T_{\Pi_5}$ and so the manifold is coisotropic. Notice that embedding Π_5 in $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ does not change this conclusion.

While the definitions above are rather simple, in order to describe a coisotropic A-brane it is more convenient to use an alternative, equivalent definition of coisotropic submanifold. Let us first define the symplectic orthogonal bundle of $\mathcal{N}$ as

$$(T_{\mathcal{N}})^J \,=\, \{Y \in T_{\mathcal{M}} \mid Y^\rho J_{\rho\sigma} X^\sigma = 0,\, \forall X \in T_{\mathcal{N}}\} \tag{A.4}$$

and then define a coisotropic submanifold whenever $(T_{\mathcal{N}})^J \subseteq T_{\mathcal{N}}$. Because J is non-degenerate, this inclusion can only be an equality when $\mathcal{N}$ is middle-dimensional with respect to $\mathcal{M}$, and in that case we are dealing with a Lagrangian submanifold. For the same reason, we can define the quotient bundle $T_{\mathcal{N}}/(T_{\mathcal{N}})^J$. In our example above, it is easy to see that $(T_{\mathcal{N}})^J = \langle X^1 \rangle$, and hence $T_{\mathcal{N}}/(T_{\mathcal{N}})^J = T_{(\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3}$.

While the first supersymmetry condition for an A-brane is that it wraps a coisotropic submanifold, the other two involve the worldvolume flux $\mathcal{F}$ and they read:

- $\mathcal{F}$ has no components on $(T_{\mathcal{N}})^J$, so it can be defined as a 2-form living in $T_{\mathcal{N}}/(T_{\mathcal{N}})^J$

- $J^{-1}\mathcal{F}$ defines a complex structure on $T_{\mathcal{N}}/(T_{\mathcal{N}})^J$

Finally, from these conditions one can show that $T_{\mathcal{N}}/(T_{\mathcal{N}})^J$ has even complex dimension, and that if $\dim(\mathcal{M}) = 2n$, then $\dim(\mathcal{N}) = n + 2k$, $k \in \mathbb{N}$.

Let us check that those conditions are also satisfied for our canonical example (3.5). It is clear that $\mathcal{F} = B|_{\Pi_5} + 2\pi\alpha' F$ has no components on $(T_{\Pi_5})^J$, so it only remains to check

that $J^{-1}\mathcal{F}$ is a complex structure in $T_{\mathcal{N}}/(T_{\mathcal{N}})^J = T_{(\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3}$. Applying the definitions above we have that

$$
J^{-1}\mathcal{F} = \begin{pmatrix} -\frac{\mathrm{Im}\,T_2}{\mathrm{Re}\,T_2} & 0 & 0 & -\frac{1}{\mathrm{Re}\,T_2} \\ 0 & -\frac{\mathrm{Im}\,T_2}{\mathrm{Re}\,T_2} & -\frac{1}{\mathrm{Re}\,T_2} & 0 \\ 0 & -\frac{1}{\mathrm{Re}\,T_3} & -\frac{\mathrm{Im}\,T_3}{\mathrm{Re}\,T_3} & 0 \\ -\frac{1}{\mathrm{Re}\,T_3} & 0 & 0 & -\frac{\mathrm{Im}\,T_3}{\mathrm{Re}\,T_3} \end{pmatrix}
\tag{A.5}
$$

and so it is easy to see that $(J^{-1}\mathcal{F})^2 = -\mathbf{1}_4$ if and only if $\mathrm{Im}\,T_2 T_3 = 0$ and $\mathrm{Re}\,T_2 T_3 = 1$, which is precisely the F-flatness condition in (3.6).

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